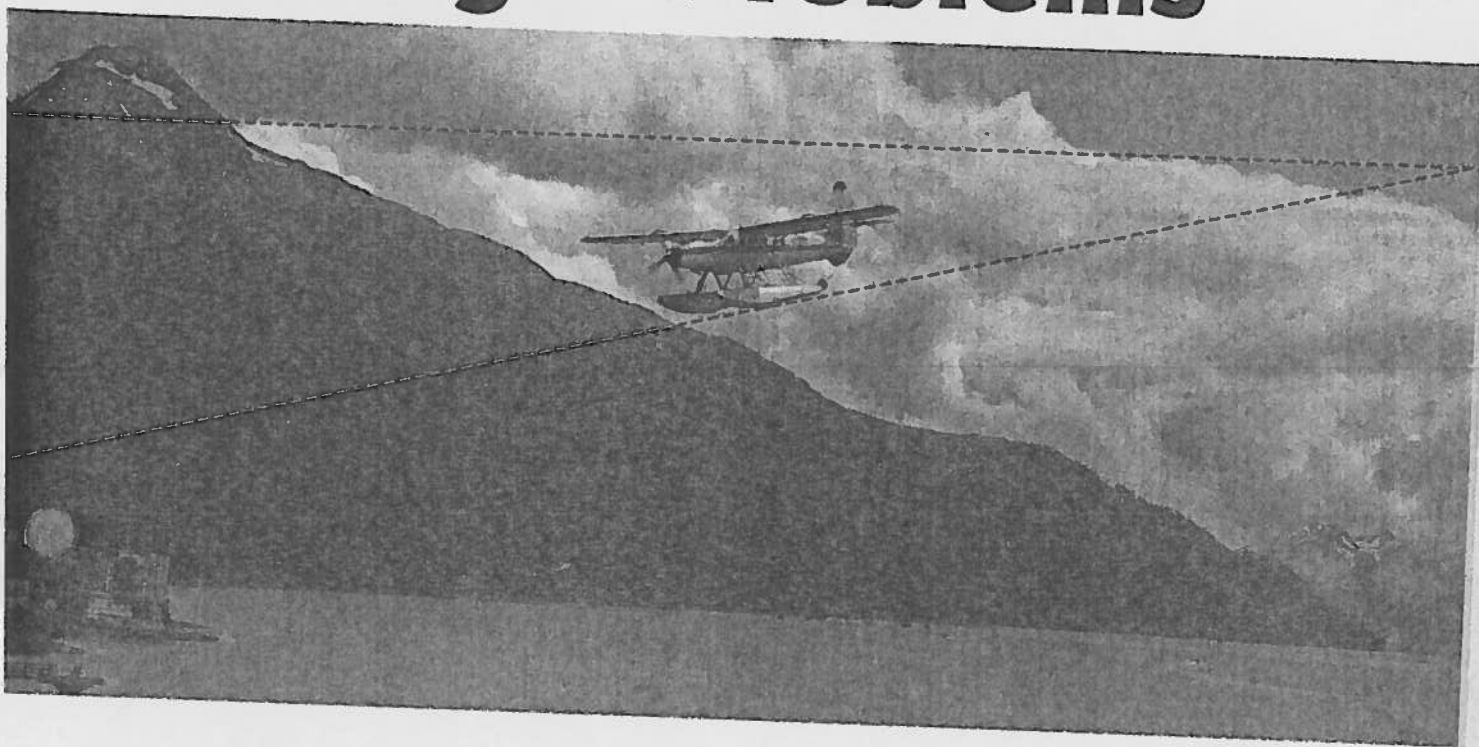# Solving Right Triangle Problems

9

Grant is a seaplane pilot in British Columbia. When he lands, he needs to know exactly where the plane will touch down.

A. How can knowing about angles and **trigonometry** help Grant land his plane?

_____

_____

_____

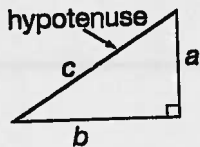**B.** How else might a pilot use trigonometry during a flight?

_____

_____

_____

9
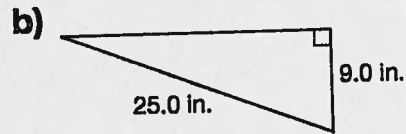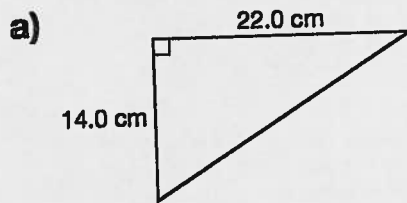
Getting Started

Hint

The Pythagorean theorem is

$a^2 + b^2 = c^2$

hypotenuse

1. Use the Pythagorean theorem to calculate the unknown side length. Label each length, to one decimal place.

a)
22.0 cm

14.0 cm

b)
9.0 in.

25.0 in.

$(14.0 \text{ cm})^2 + (22.0 \text{ cm})^2 = c^2$

$\underline{\quad} \text{ cm}^2 + \underline{\quad} \text{ cm}^2 = c^2$

$\underline{\quad} \text{ cm}^2 = c^2$

$\underline{\quad} \text{ cm} = c$

$(9.0 \text{ in.})^2 + b^2 = (25.0 \text{ in.})^2$

$b^2 = (25.0 \text{ in.})^2 - \underline{\quad}$

$b^2 = \underline{\quad} \text{ in.}^2$

$b = \underline{\quad} \text{ in.}$

Hint

sin A means "sine of ∠A."

The three trigonometric ratios for ∠A are

$\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$

2. What is each trigonometric ratio for the triangle on the right, as a fraction and as a decimal?

B
3
5
C
4
A
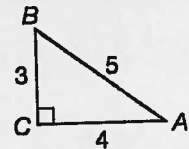
a) $\sin A = \dfrac{3}{5}$, or ____

b) $\cos A = \dfrac{\square}{\square}$, or ____

c) $\tan A = \dfrac{\square}{\square}$, or ____

d) $\sin B = \dfrac{\square}{\square}$, or ____

e) $\cos B = \dfrac{\square}{\square}$, or ____

f) $\tan B = \dfrac{\square}{\square}$, or ____

3. Calculate.

a) $4 \times \sin 25° =$ _____

b) $\dfrac{16}{\cos 56°} =$ _____

Tech Tip

Determining Unknown Angles

If you know the **sine**, **cosine**, or **tangent** of an angle, you can use the 2nd sin, 2nd cos, or 2nd tan keys to determine the angle measure. Make sure your calculator is in Degree mode.

Sin S = 0.5. To determine ∠S, enter 2nd sin 0.5 =. Your answer should be 30°. If it is not, try 0.5 2nd sin =.

4. What is the measure of each angle, to the nearest degree?
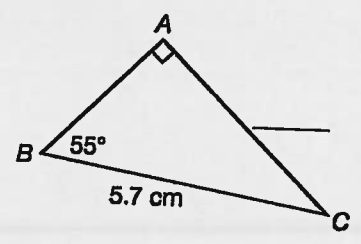
a) $\sin C = 0.8387$

$\sin^{-1}(0.8387) = \underline{\hspace{2cm}}$

$\angle C \doteq \underline{\hspace{2cm}}$

b) $\tan F = \dfrac{7}{3}$

$\tan^{-1}\left(\dfrac{7}{3}\right) = \underline{\hspace{2cm}}$

$\angle F \doteq \underline{\hspace{2cm}}$

5. How long is each side, to one decimal place? Label the length on the diagram.

a) side CA

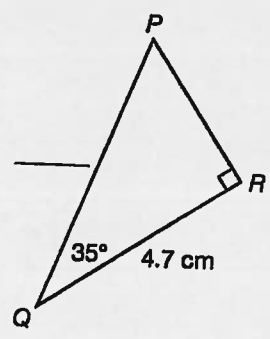$\sin 55° = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$\sin 55° = \dfrac{CA}{\Box}$

$\underline{\hspace{2cm}} \times \sin 55° = CA$

$\underline{\hspace{2cm}} = CA$

So $CA = \underline{\hspace{2cm}}$ cm, to one decimal place.

b) side PQ

$\cos 35° = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$\cos 35° = \dfrac{\Box}{PQ}$

$PQ \times \cos 35° = \underline{\hspace{2cm}}$

$PQ = \dfrac{\Box}{\cos 35°}$

$PQ = \underline{\hspace{2cm}}$

So $PQ = \underline{\hspace{2cm}}$ cm, to one decimal place.
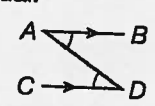
6. What are the unknown angle measures in each triangle? Mark the angle measurements on the diagrams.

a)

b)
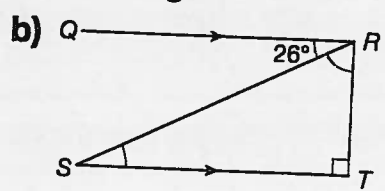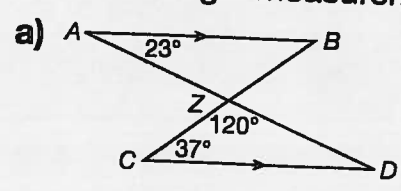
9.1 Angles of Elevation
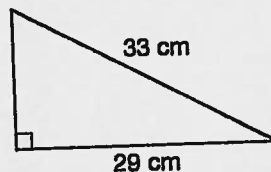
Try These

You will need
• a ruler

i) Which is the measure of the angle of elevation?

65°

25°
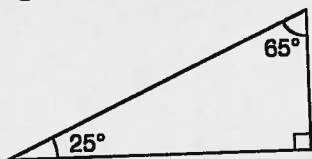
Angle of elevation = _____

ii) Write an expression to use for calculating the angle of elevation.

33 cm

29 cm

Angle of elevation = _____

Arya builds roof trusses for a construction company in Medicine Hat. He needs to cut the ridge beam at an angle to attach it to the rafter.

rafters

cut angle

6 ft

$x°$

12 ft

ridge beam

The cut angle must match the angle of elevation of the rafter, or $x°$. At what angle should Arya cut the ridge beam?
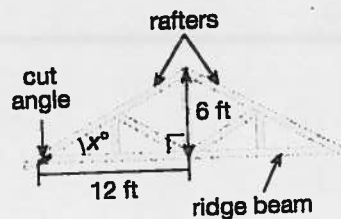
REFLECTING
How did you decide whether to use a sine, cosine, or tangent ratio to solve the problem?

1. What equation can you use to calculate $x°$?

$$\underline{\hspace{1cm}} x° = \frac{\square}{\square}$$

$$x° = \underline{\hspace{0.6cm}}^{-1}\left(\frac{\square}{\square}\right)$$
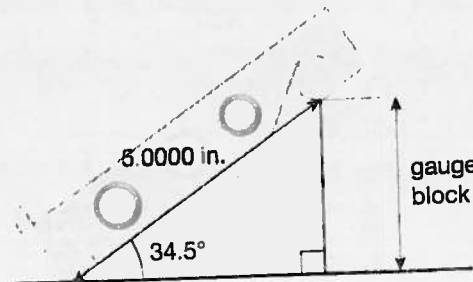
2. What is the cut angle, to the nearest degree? $x° = $ _____

Arya should cut at an angle of about _____.

Example 1

Shaina is a machinist. She uses a sine bar and gauge block to measure angles very accurately. For this project, the angle of elevation must be exactly 34.5°.

5.0000 in.

gauge block

34.5°

Which gauge-block height should she use? Express your answer to 4 decimal places.

Solution

A. What is an equation for the gauge-block height, h?

$$\underline{\hspace{1.5cm}} \; 34.5° = \frac{h}{\boxed{}}$$

$$\underline{\hspace{1.5cm}} \times \underline{\hspace{1.5cm}} = h$$

B. The height of the gauge block should be _____ in.

Example 2

Vanessa needs to build a wheelchair ramp for a cottage in Whiteshell.
- The ramp must meet the porch 0.35 m above the ground.
- The distance along the ground from the start of the ramp to the base of the porch must be 4.25 m.

What angle of elevation and ramp length should Vanessa use?

Solution

A. Sketch a right triangle to show a side view of the ramp. Label the lengths of the **legs**. Use $x°$ to represent the angle of elevation and r to represent the ramp length.

legs
the two sides that form the 90° angle in a right triangle

B. What is the ramp's angle of elevation, to the nearest degree?

$$\underline{\hspace{1.5cm}} x° = \frac{\boxed{}}{\boxed{}}$$

$$x° = \underline{\hspace{0.7cm}}^{\boxed{}}\!\left(\frac{\boxed{}}{\boxed{}}\right)$$

$$x° = \underline{\hspace{1.5cm}}°$$

The angle of elevation should be about _____.

C. Is your answer reasonable? Explain.

_____

REFLECTING
According to the building code, the slope of a wheelchair ramp must not be steeper than $\frac{1}{12}$. Does this ramp meet the code? Explain.

D. Use the Pythagorean theorem. Calculate the ramp length, to two decimal places.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{1cm}}$$

$$\underline{\hspace{2cm}} = \underline{\hspace{1cm}}$$
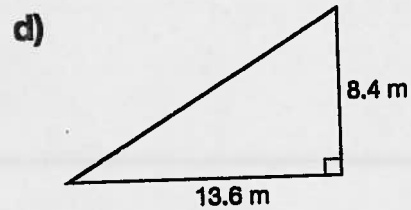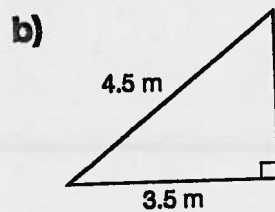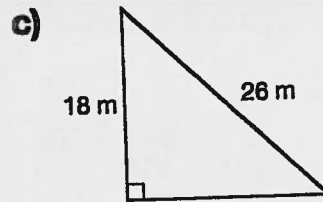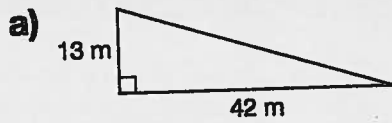
$$\underline{\hspace{2cm}} = r$$

The ramp length should be _____, to two decimal places.

Practice

1. Label the angle of elevation, $x°$, on each diagram. What is the measure of each angle of elevation, to the nearest degree?

a)

13 m, 42 m

c)

18 m, 26 m

b)

4.5 m, 3.5 m

d)

8.4 m, 13.6 m

2. Record the given angle of elevation on each diagram. What is each unknown side length, to one decimal place?

a) Angle of elevation = 21°

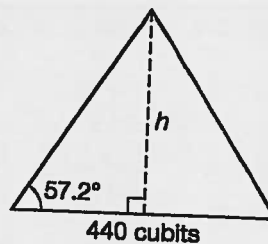w, 5.6 m, x

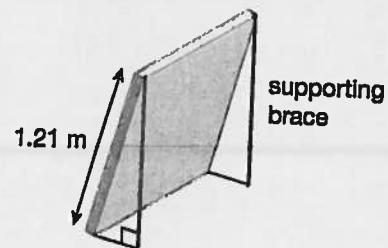b) Angle of elevation = 43°
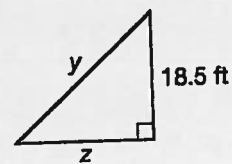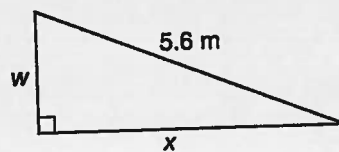
y, 18.5 ft, z

3. This diagram shows one face of the Great Pyramid of Giza. The pyramid is one of the seven wonders of the ancient world. What is the slant height, h, of the face, in cubits?

Hint
A cubit is an ancient unit of length. It was based on the distance from the elbow to the tip of the middle finger.

4. Bailey installs solar panels in Saskatoon. She adjusts the angle to match the season. In winter, the angle of elevation for panels in Saskatoon is 75.8°. The panels are 1.21 m tall. The roof is flat. How tall should the supporting brace be?

5. In spring and fall, the panels from Question 4 will tilt to meet the brace at a height of 0.92 m. Draw a diagram. What will the angle of elevation be for spring and fall?

6. The statue of Smokey the Bear is a landmark in Revelstoke. At 6.3 m from the base, the angle of elevation to the top is 55°. Draw a diagram. What is the height of the statue?
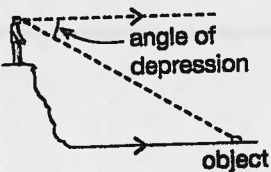
9.2 Angles of Depression

Try These

i) Name the **angle of depression** in the diagram.

∠_____

ii) What is the angle of depression? _____

angle of depression

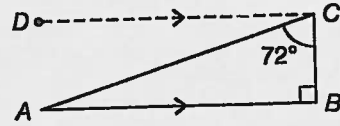the angle between the horizontal and the line of sight when looking *down* at an object

angle of depression

object

Hint
The angles in a triangle have a sum of 180°.

A search-and-rescue helicopter spots a lost hiker at an angle of depression of 31°. The helicopter is flying 1100 ft directly above a ground rescue crew.
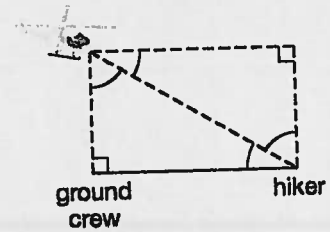
How far, to the nearest foot, is the ground crew from the hiker?

ground crew   hiker

① Label the helicopter height and the angle of depression on the diagram. Label the distance from the crew to the hiker x ft.

② Use the given angle to calculate the measures of the other angles. Label these on the diagram.

③ What equation can you use to determine the distance from the crew to the hiker?

$$\underline{\hspace{2cm}} = \frac{\boxed{}}{\boxed{}}$$

$$\underline{\hspace{1cm}} \times \underline{\hspace{1.5cm}} = \underline{\hspace{1.5cm}}$$

$$x = \frac{\boxed{}}{\boxed{}}$$

REFLECTING
How could you solve this problem a different way?

④ How far from the hiker is the ground crew?

x ft = _____ ft

The crew needs to go _____ ft to reach the hiker.

⑤ Is your answer reasonable? Explain.

_____

_____

Example 1

Luc, *L*, is on a cliffside trail above a river. He sees his brother Marcel, *M*, at ground level on the far shore. The horizontal distance between them is 390 m.

Luc is looking down at an angle of depression of 72°. About how high is the cliff?
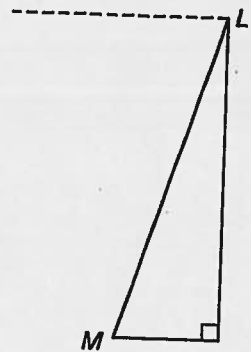
Solution

A. Label the given lengths and angles on the diagram. Label the height of the cliff *h* m.

B. What is the angle of elevation for Marcel? _____ Add this to the diagram.

C. The equation for the height, *h*, of the cliff is

$$\underline{\hspace{2cm}} \, 72° = \frac{\square}{\square}$$

$$\underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}} = h$$

The cliff is about _____ high.

Example 2

A pilot begins her final descent to an airport from 300 m above the ground. At this point, her horizontal distance to the airport is 5.7 km. What is the angle of depression for her path of descent?

Solution

A. For an equation, lengths are written with the same unit. Express 5.7 km in metres: 5.7 km = _____ m

B. Draw a diagram of the plane's descent. Label the known distances. Label the angle of depression *x*°.

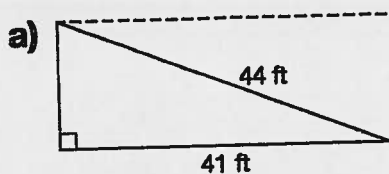C. What is an equation for determining the value of *x*°?

$$\underline{\hspace{2cm}} \, x° = \frac{\square}{\square}$$

$$x° = \underline{\hspace{2cm}}^{-1}\left(\frac{\square}{\square}\right)$$

D. The angle of depression is about _____.

Practice

1. What is the measure of each angle of depression? Label this measure on the diagram.

a)

44 ft

41 ft

$x° =$ _____ $^{-1}$ (▱)

The angle of depression is about _____.

b)

13.9 m

18.4 m

$x° =$ _____ $^{-1}$ (▱)
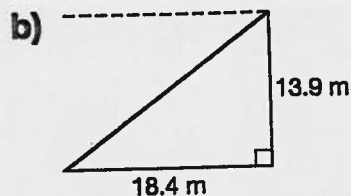
The angle of depression is about _____.

2. What are the unknown side lengths, to one decimal place?

a)

61°

6.4 cm

x

w

b)

18°

8.2 in.

y

z

light

$x°$

7.5 m

P

9.0 m
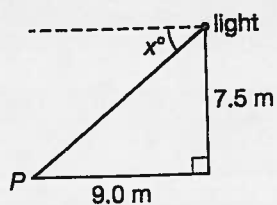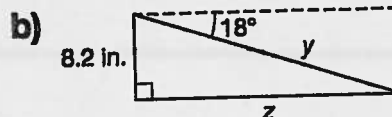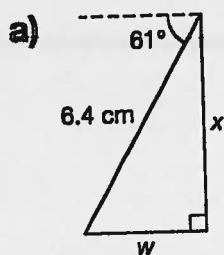
3. Fiona works for a theatre lighting crew. A light has to be aimed at an area shown by point P in the diagram. At what angle of depression should Fiona aim the light?

4. William and Pam are ride operators for a travelling midway. In the next city, the roller coaster is to be set up inside a domed stadium 56 m high.

To determine the height of the ride, William rides to the top. Pam stands 13 m from the bottom and looks up 73° to see William.

a) At what angle of depression would William look down from the top to see Pam? _____

b) Pam's eyes are 1.5 m above the ground. Is the roller coaster too tall for the stadium?

William

Pam
1.5 m
73°
13.0 m
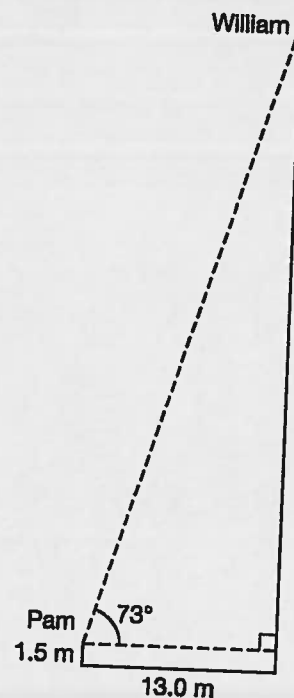
5. A fire tower in Alberta is 120 ft high. An observer in the tower spots a bear at an angle of depression of 21°.

Draw a diagram to represent the situation. How far from the tower base is the bear? Give your answer to the nearest foot.

6. How can knowing about angles of elevation help you solve problems about angles of depression?
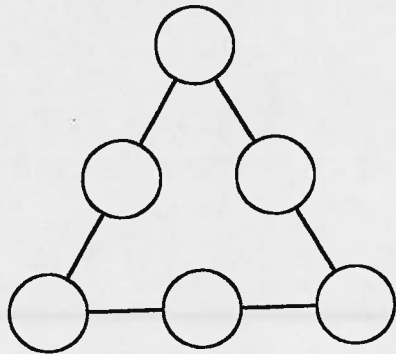
9.3 Solving Triangle Puzzles

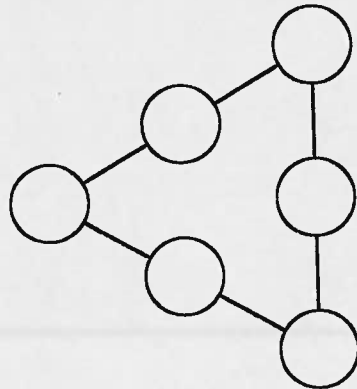Arrange the numbers 1, 2, 3, 4, 5, and 6 around each triangle to get the sum shown for each side. Use each number only once per triangle. Explain your strategy.
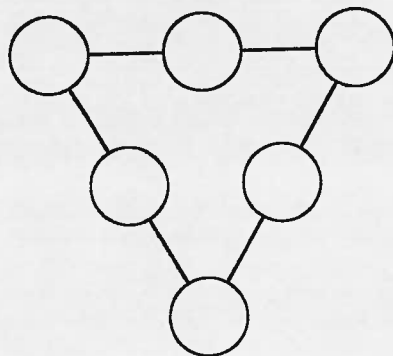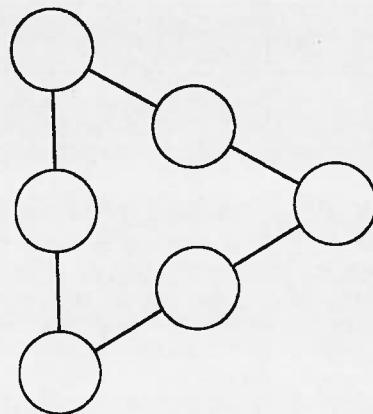
A. Sum for each side: 9

C. Sum for each side: 11

B. Sum for each side: 10

D. Sum for each side: 12

Mid-Chapter Review

1. What is the value of x, to one decimal place?

    a) $\sin 72° = \dfrac{x}{28}$

    b) $\cos 35° = \dfrac{8}{x}$

2. Aiko is flying a kite. She is using a string that is 95 ft long. How high above the ground is the kite?
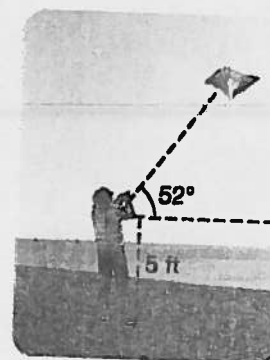
3. Yanek builds custom decks. The stairs for a customer have a riser of 160 mm and tread of 290 mm. What is the angle of elevation, $x°$, of the stringer, to the nearest degree?
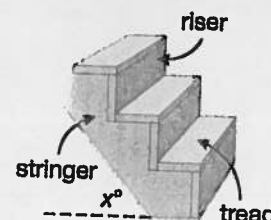
4. An observer is on the Calgary Tower Observation Deck, 157.5 m above the ground. She estimates the angle of depression to a nearby building as 40°. The building is 61 m away from the Calgary Tower. How tall is the building?
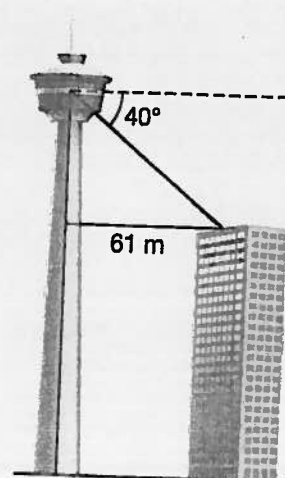
9.4 Solving Two-Triangle Problems

Try These

i) Name all the right triangles in the diagram.

_____

ii) PR = 25 cm and QR = 16 cm.
So PQ = _____ cm.

Nuvuk works as a tour guide in Rankin Inlet. He often takes pictures of the tourists by the giant inuksuk.

• Nuvuk took a picture of Milan. Milan is 187 cm tall.
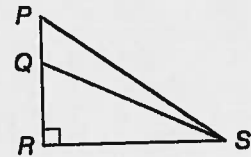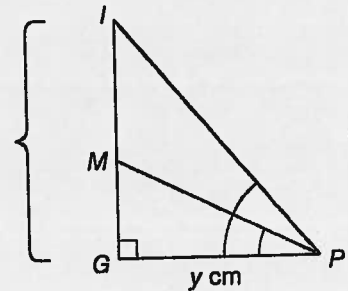• From point P, the angle of elevation to the top of Milan's head was 26°.
• The angle of elevation to the top of the inuksuk was 50°.

How tall is the inuksuk, to the nearest tenth of a metre?

Hint
MG shows Milan's height. IG shows the height of the inuksuk.

1 Label the diagram to show the known angles and side lengths. Label the inuksuk height x cm.

2 What is the distance from P to the centre of the base of the inuksuk?

$$\underline{\hspace{1cm}} = \frac{\square}{\square}$$

$$\underline{\hspace{0.5cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{0.5cm}}$$

$$y = \frac{\square}{\boxed{}}, \text{ or } \underline{\hspace{1cm}}$$

The distance is _____ cm.

REFLECTING
Is your final answer reasonable? Explain.

3 How tall is the inuksuk?

$$\underline{\hspace{1cm}} = \frac{\square}{\boxed{}}$$

$$\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = x$$

$$\underline{\hspace{1cm}} = x, \text{ or } x \doteq \underline{\hspace{1cm}} \text{ cm}$$

The inuksuk is _____ tall, to the nearest tenth of a metre.

NEL

Example

A camp instructor leads a group of students on a canoe trip across Sylvan Lake.

- They leave Half Moon Bay, H. They paddle 7.8 km to Jarvis Bay Provincial Park, J.
- Then they turn 69°. They paddle 5.1 km to a beach. This beach, B, is 7.6 km across the lake from Half Moon Bay.

At what angle should they turn in order to return to where they started?
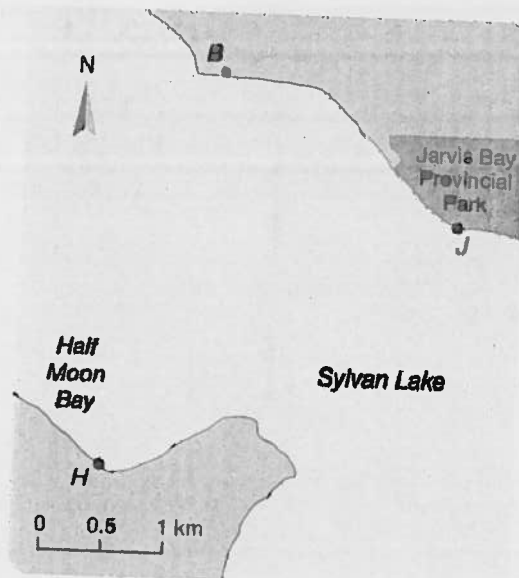
Solution

A. Draw the triangle that maps their route. Label the lengths and angles. Is △HJB a right triangle? Show how you know.

B. Draw a height, h, from H to point X on BJ. What kind of triangles are △HXJ and △HXB? Show this on the diagram.

_____

C. Calculate the length of h.

$$\underline{\quad\quad} \, 69° = \frac{h}{\boxed{}}$$

D. At what angle should the canoeists turn?

$$\underline{\quad\quad} \, B = \frac{\boxed{}}{7.6}$$

$$\angle B = \underline{\quad\quad}{}^{-1}\left(\frac{\boxed{}}{7.6}\right)$$

$$\angle B = \underline{\quad\quad}$$

They should turn at an angle of about _____.

Practice
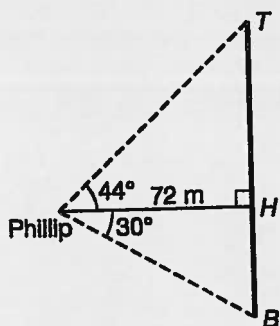
1. Phillip is on the shore looking at a pillar, *BT*, on the Lion's Gate Bridge in Vancouver.

(Diagram: Triangle with Phillip at left vertex. From Phillip, a 44° angle above a 72 m horizontal line to point H (right angle at H), with T above H. Below, a 30° angle down to point B.)

a) What is the height of each part of the pillar?

From *H* to *T*:   From *B* to *H*:

b) How many metres tall is the pillar from bottom to top?

2. Your angle of peripheral vision determines how far you can see from left to right when you look straight ahead. An average person sees about 120°.

(Diagram: Horizontal line A to J measuring 9.2 m, with X at top center, point C below with 3.0 m down from X. Angle at C marked.)

- To calculate his angle of peripheral vision, Cal asks Julian and Ann to walk in opposite directions from point X. He watches from point C, 3.0 m away from X.
- Julian and Ann stop when Cal can no longer see them. Julian and Ann have each walked the same distance.
- The final distance from Ann to Julian is 9.2 m.
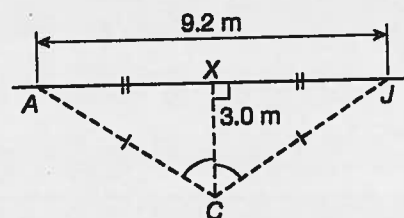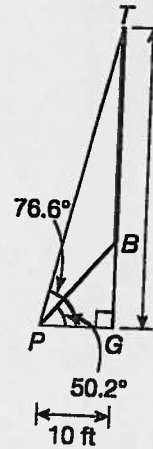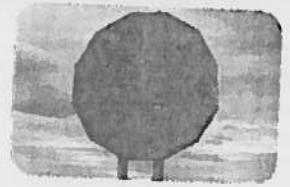
What is Cal's angle of peripheral vision, ∠*ACJ*?

3. The Big Nickel in Sudbury is the world's largest coin.
 • From point P, 10 ft away from the coin's base on the ground, the angle of elevation to the bottom of the nickel is 50.2°.
 • The angle of elevation to the top of the nickel is 76.6°.

a) What is the distance from the ground, G, to each point?

Height to bottom of nickel, B:

Height to top of nickel, T:

b) What is the height of just the nickel, to the nearest foot?

4. Anya is an aerial photographer. She plans to photograph the Lion's Gate Bridge from a helicopter.
 • The bridge is 1517 m long.
 • The helicopter will be 370 m above and 600 m away horizontally from one end of the bridge.

Can Anya capture the entire distance across the bridge with a 140° wide-angle lens? Draw a diagram with your solution.
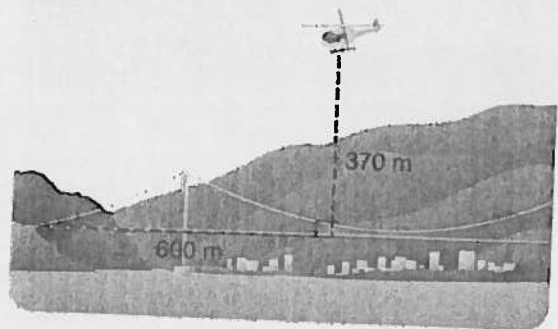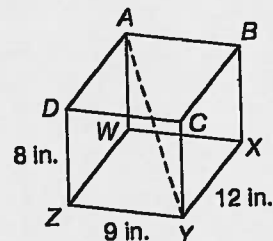
9.5 Solving 3-D Triangle Problems

Try These

Draw a line to form two triangles that you could use to determine the length of diagonal, *AY*, of the box. Name the triangles.

_____

_____

_____

Carl and Devon are rappelling instructors. They need to determine the height of the second peak of Stawamus Chief, from the base, *B*, to the top, *A*.

• From Carl, *C*, the angle of elevation to the cliff top, ∠*ACB*, is 51°.

• Devon, *D*, walks 105 m away from Carl, at a right angle to *BC*. ∠*BDC* is 78°.

How high is the cliff, to the nearest metre?
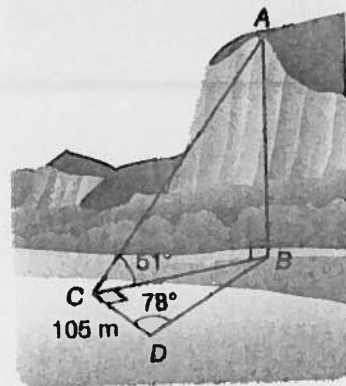
① Name the right triangles in the diagram.

△_____ and △_____

② How far is it from where Carl is standing at *C* to the base of the cliff at *B*?

$$\tan \text{\_\_\_\_\_} = \frac{CB}{\square}$$

_____ × _____ = CB

_____ = CB

③ How high is the cliff, to the nearest metre?

$$\text{\_\_\_\_\_} = \frac{AB}{\square}$$

_____ × _____ = AB

_____ = AB

The cliff is _____ high, to the nearest metre.

Example

Ranaan works in the Langara Point Lighthouse. The top of the lighthouse is 160 ft above sea level. Ranaan sees a boat in distress at D. He sees a Coast Guard boat at C.

How far apart are the two boats?
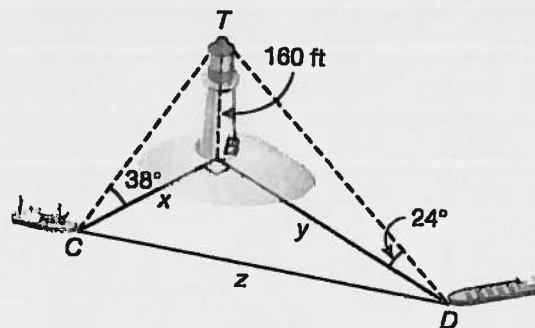
Solution

A. Name all the right triangles in the diagram.

_____

B. Calculate the horizontal distance from point B to each boat.

Coast Guard boat:

$$\tan \underline{\quad} = \frac{160}{\square}$$

$$x \times \tan 38° = 160$$

$$\underline{\quad} = \frac{\square}{\square}$$

$$\underline{\quad} = \underline{\quad\quad}$$

Boat in distress:

$$\tan \underline{\quad} = \frac{160}{\square}$$

$$y \times \tan 24° = 160$$

$$\underline{\quad} = \frac{\square}{\square}$$

$$\underline{\quad} = \underline{\quad\quad}$$

C. Calculate the distance between the two boats.

$$z^2 = x^2 + y^2$$

$$z^2 = \underline{\quad\quad} + \underline{\quad\quad}$$
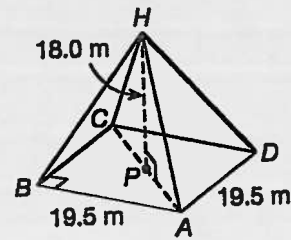
$$z^2 = \underline{\quad\quad}$$

$$z = \underline{\quad\quad}$$

The boats are about _____ ft apart.

Practice

1. A pyramid at the Muttart Conservatory in Edmonton has a square base with 25.70 m sides and is 24.00 m high. What is the slant height, HX? Mark the measurements on the diagram.
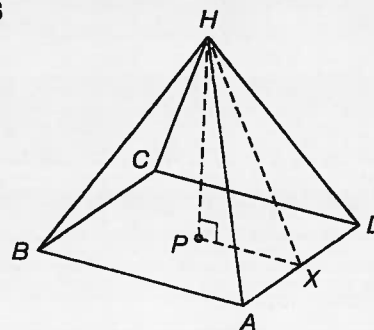
2. The Muttart Conservatory has four pyramids. Two pyramids have a square base with sides 19.5 m long. They are 18.0 m high.

Judy calculated the angle of elevation of each face of the pyramid. Is her solution reasonable?

$$AB^2 + BC^2 = AC^2$$

$$19.5^2 + 19.5^2 = AC^2$$

$$27.577... = AC, \text{ or } AC = 27.577... \text{ m} \quad AP \text{ is half of } AC.$$

Length of AP: $\dfrac{27.577...}{2} = 13.788..., \text{ or } 13.788... \text{ m}$

To get $\angle HAP$: $\cos \angle HAP = \dfrac{13.788...}{18.0}, \text{ or } 40.001...°$

The angle of elevation is about 40°.

a) Was the length Judy calculated for AC reasonable? Explain.

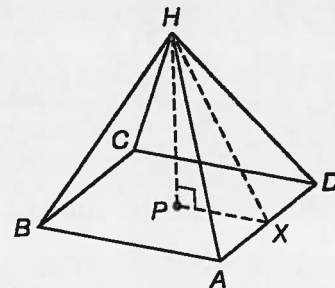_____

_____

b) Did Judy use the correct trigonometric ratio? Explain.

_____

_____

c) Was Judy correct to determine $\angle HAP$? Explain.
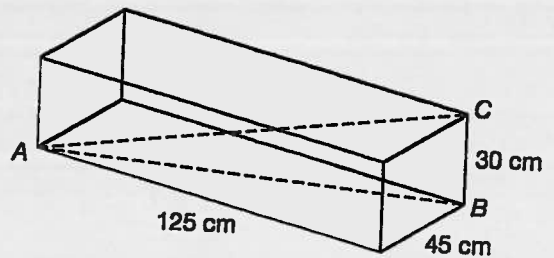
_____

_____

3. What is the angle of elevation for the pyramid in Question 2, to one decimal place?

Mark the measurements on the diagram.

4. a) Ryan runs an online parts supply business. What is the longest piece of thin copper tubing he can pack in the box shown? Assume that the tubing does not bend.

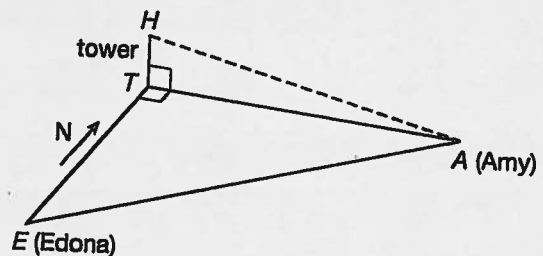Diagonal length *AB*:   Inside diagonal *AC*:

A   *C*   30 cm   *B*   125 cm   45 cm

b) What is the measure of ∠*CAB* in Part a)?

5. A communications tower lies due north of Edona's house. Edona walks 1.5 km to Amy's house, travelling 40° east of north. The tower is due west of Amy's house.

The angle of elevation from Amy's house to the top is 10°.

H   tower   *T*   *N*   *A* (Amy)   *E* (Edona)

a) How tall is the tower, to the nearest metre?

b) Is the angle of elevation from Edona's house to the top of the tower more than, less than, or equal to 10°? Explain.
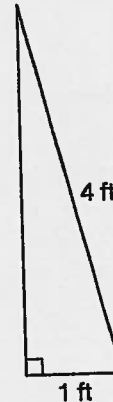
Chapter Review

1. Brynn installs soffit, fascia, and eavestrough. Safety regulations state that the base of a ladder must be at least 1 ft away from the wall for every 4 ft of ladder length.

   a) What is the maximum safe angle of elevation for a ladder, without rounding?

4 ft

1 ft

   b) Brynn's ladder is 24 ft long. How high can a 24 ft ladder reach, without rounding?

2. The world's largest dinosaur, in Drumheller, is 86 ft tall. Anita measured the angles of elevation to its head and its tail from points on the ground. Each point was the same horizontal distance from the dinosaur, x ft. How high is the tail, y to the nearest foot?

86 ft

56°

x

37°

y

x
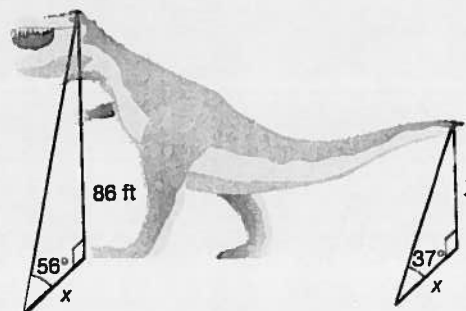
3. Alexis installs wind turbines on the Prairies. For installations on rocky ground, she uses guy wires to secure the pole.
   - The wires are anchored at a point 24 ft from the pole base.
   - The wires meet the pole at 20 ft and 40 ft above the ground.

   Draw a diagram to show the angle of depression of each wire. Label the lengths and angles you know. What is the angle of depression of each wire, to the nearest degree?
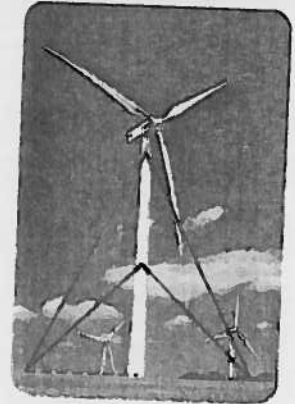
4. To construct the Esplanade Riel Pedestrian Bridge in Winnipeg, surveyors needed to determine the width across the river.

   The diagram shows $\triangle CDE$ measured out on shore BD. How wide is the river, w, to the nearest metre?
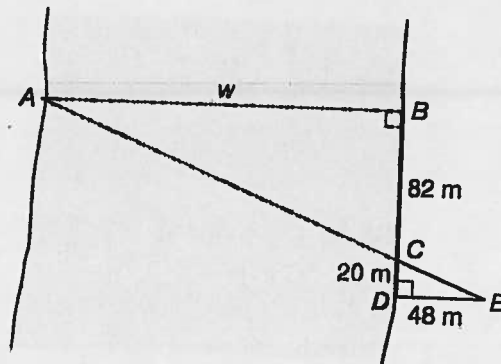
5. How are the angles of elevation and angles of depression similar? How are they different?

Chapter Test

1. Brooke replaces old wooden utility poles with steel ones.
   - Brooke stands between two poles. She is 25 ft from the base of Pole 1.
   - The angle of elevation from Brooke's feet to the top of Pole 1 is 54°.
   - A linesman is on top of Pole 2. The angle of depression to Brooke's feet is 19°.

   a) Draw a diagram for the problem. How tall is Pole 1?

   b) The two poles are the same height. How far apart are the poles, to the nearest foot?

2. Phuong is installing a zip line along the line *BD*. It must land at a distance of 200 m below the starting point on the far side. How long is the zip line, to the nearest metre?