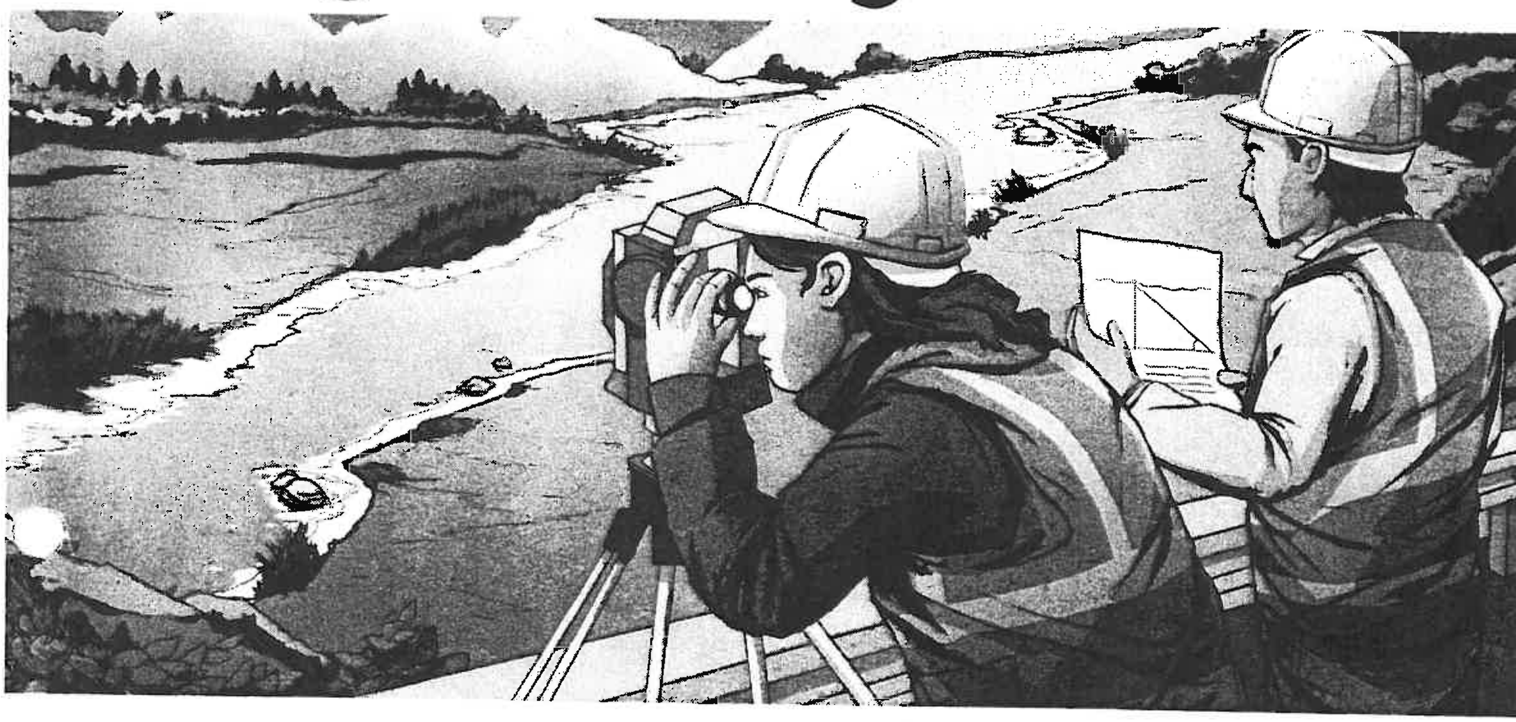
Relationships in Right Triangles

8



These surveyors are measuring the distance across a river so that a new bridge can be built. Surveyors use **trigonometry** in their work. To measure long distances, they might use a laser measuring device. To measure horizontal and vertical angles, they might use a theodolite—an instrument made by combining a telescope and two circular protractors.

trigonometry
the study of relationships among the sides and angles in right triangles

A. Why do you think surveyors need special tools to measure angles and distances outside?

____

____

B. Think of another situation outdoors where a surveyor might be needed to make measurements.

____

____

8 Getting Started

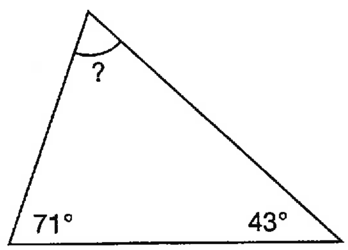
You will need
- a scientific calculator

(Trig functions are needed in most of this chapter.)

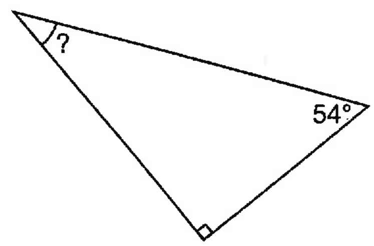
Hint
The sum of the angles in a triangle is 180°.

1. What is the complement of each angle?
   a) 23° 90° − 23° = ______
   b) 45° ______
   c) 78° ______
   d) 51° ______

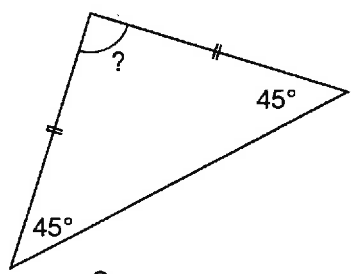
2. Calculate the measure of each indicated angle.

a)


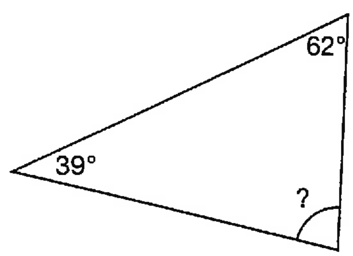
? = ______

c)


? = ______

b)


? = ______

d)


? = ______

3. Use the triangles in Question 2.
   a) Is the longest side opposite the largest angle? ______
   Mark it on each triangle with a checkmark.
   b) Which triangles are right triangles? ______

square number
the result when a whole number is multiplied by itself

4. Complete the list of **square numbers** from 1^2 to 13^2.
   1, 4, 9, ______, ______, ______, ______, ______, ______, ______, ______, ______, ______

5. Calculate the square root. (The first one is done for you.)
   a) $\sqrt{2}$ = 1.414...
   b) $\sqrt{4}$ = ______
   c) $\sqrt{81}$ = ______
   d) $\sqrt{0.36}$ = ______
   e) $\sqrt{6.76}$ = ______
   f) $\sqrt{5}$ = ______

6. What is the inverse operation?

a) The inverse of $\boxed{-}$ is $\square$.   b) The inverse of $\boxed{x^2}$ is $\square$.

7. Calculate.

a) $7.5^2 + 18^2 =$ ________   b) $19.5^2 - 7.5^2 =$ ________

8. Compare. Write = or ≠.

a) $5^2 \square 3^2 + 2^2$ c) $12 \square 4^2 - 2^2$

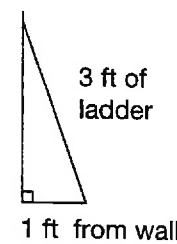
b) $4^2 \square (5^2 - 3^2)$ d) $50 \square \sqrt{(36 + 64)}$

9. Solve for x. Check by substituting your answer into the equation.

a) $2x = 24$, so $x =$ ______   c) $x = \sqrt{121}$, so $x =$ ______

b) $\frac{x}{3} = 15$, so $x =$ ______   d) $\sqrt{169} = 2x$, so $x =$ ______

10. For safety, a ladder should be at least 1 ft from the wall for every 3 ft of ladder. How far should the bottom of a 10 ft ladder be from the wall?



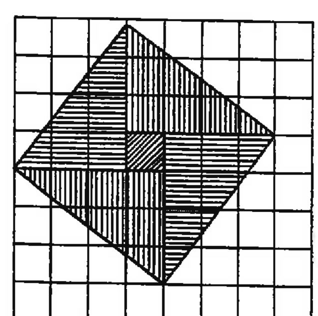
The **ratio** is $\frac{\text{distance from wall}}{\text{length of ladder}}$.

$$\frac{1 \text{ ft}}{3 \text{ ft}} = \frac{? \text{ ft}}{\square \text{ ft}}$$

A 10 ft ladder should be ______ ft from the wall.

11. a) What is the area of this composite shape?

Area = 4 × (area of triangle) + area of small square



1 square represents 1 cm^2.

b) What is the length of each side of the larger shaded square?

$(\text{length of side})^2 = \text{area of square}$

8.1 The Pythagorean Theorem

Try These

You will need
- grid paper
- a ruler
- a protractor
- scissors

Calculate. If necessary, round your answer to two decimal places.

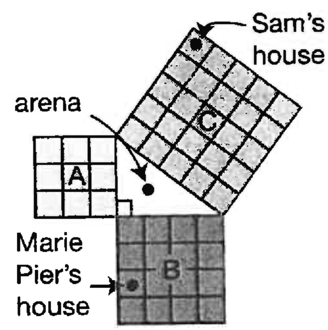
i) $21^2 =$ ______

ii) $3.2^2 + 2.3^2 =$ ______

iii) $\sqrt{12} \doteq$ ______

iv) $\sqrt{9^2 - 4^2} \doteq$ ______

Marie-Pier and Sam are doing a bottle drive for their hockey team. Marie-Pier will go to the houses in regions A and B, and Sam will cover region C.



1. Will Marie-Pier and Sam cover the same number of blocks? How do you know?

______

______

2. How many units long is the side opposite the right angle? How do you know?

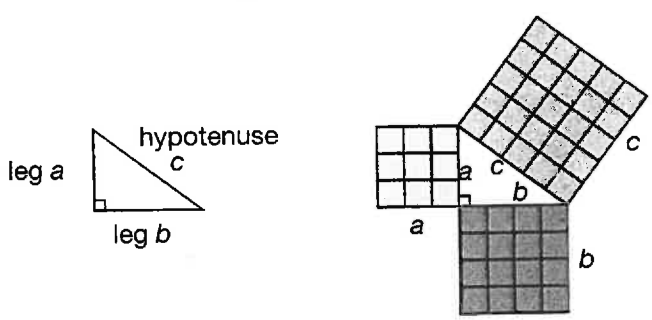
______

______

hypotenuse
the side of a right triangle that is opposite the 90° angle

legs
the two sides that form the 90° angle in a right triangle

In a right triangle, the square of the length of the **hypotenuse** is equal to the sum of the squares of the lengths of the two other sides, called **legs**.



$a^2 + b^2 = c^2$

The relationship among the sides in a right triangle, $a^2 + b^2 = c^2$, is known as the **Pythagorean theorem**. It is named after the ancient Greek mathematician Pythagoras, who proved that it was true.

Example 1

How can you show the Pythagorean theorem using a geometric model?

Solution

A. Draw a right triangle on grid paper.

B. Draw a square on each leg and on the hypotenuse, using a ruler and a protractor.

C. Cut out the squares on the legs. Then rearrange and cut these two squares as needed to fit them on the square on the hypotenuse. What do you notice?

____

D. Try it several times using a different right triangle each time. What do you notice?

____

____

E. Do you think this will be true for other kinds of triangles? Try drawing different triangles and checking the relationship. What do you notice?

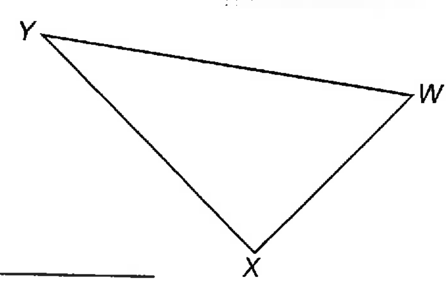
____

REFLECTING

Is the hypotenuse always the longest side in a right triangle? How do you know?

Example 2

How can you tell if ΔWXY is a right triangle, using only a ruler?



Solution

A. What are the measures of the sides? ____________________

B. What is the sum of the squares of the two shorter side lengths?

____

C. What is the square of the longest side length? ____________

D. Compare your answers for Parts B and C. What does this mean?

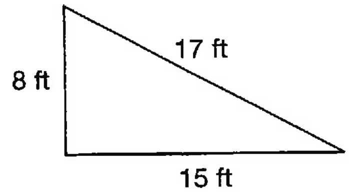
____

Practice

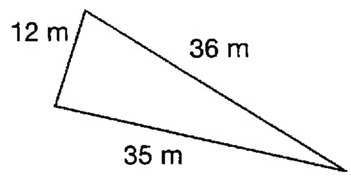
Hint
Use the formula of the Pythagorean theorem.
$a^2 + b^2 = c^2$

1. Test the Pythagorean theorem on each triangle to see which ones are right triangles.

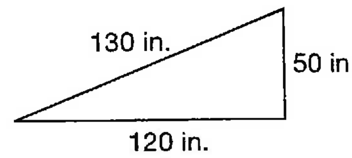
a)



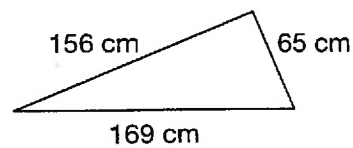
b)



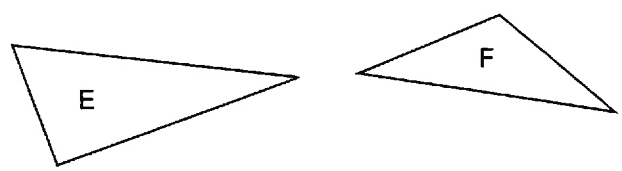
c)



d)



2. Allyson is cutting corner shelves. She wants to know which one forms a right triangle, but she only has a ruler.



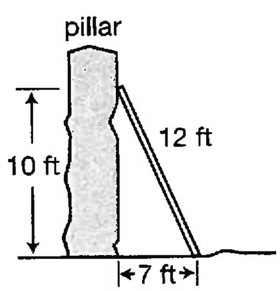
Hint
Measure in centimetres, to the nearest tenth, and record the measurements on the diagram.

a) Measure the sides of triangle E with a ruler and test the Pythagorean theorem.

____

b) Measure the sides of triangle F and test the Pythagorean theorem. ____________________________

c) Which triangle is a right triangle? __________



3. Bobby placed a 12 ft ladder against a pillar as shown.

a) What is the square of the length of the ladder? ________

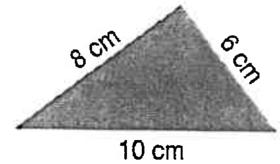
b) What is the sum of the squares of the two other measures?

c) Does the pillar form a right angle with the ground? How do you know?

____

____

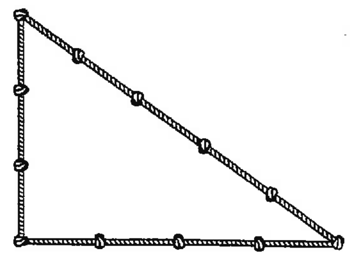
4. Zennon has cut a piece of felt to create a right triangle. Has he succeeded? Explain your thinking.



5. Are triangles X and Y right triangles? Explain.

Triangle	Longest side	Shorter sides
X	$12\frac{1}{2}$ in.	10 in., $7\frac{1}{2}$ in.
Y	2.42 m	1.20 m, 2.10 m

6. More than 4000 years ago, Egyptians used a rope with 13 equally spaced knots to build square corners in their buildings and pyramids. Explain how they knew they had a square corner. (Note that the first knot and the last knot overlap, so the triangle has 12 equally spaced knots.)



Hint
The distance between the knots doesn't have to be 1 cm or 1 in. or 1 ft. It can be any measurement. It's the **ratio** of leg : leg : hypotenuse that's important.

7. A set of three whole numbers that satisfy $a^2 + b^2 = c^2$ is called a Pythagorean triple. Test to see whether each set of numbers is a Pythagorean triple.

a) 2, 4, 6 ____________________

b) 12, 9, 15 ____________________

c) 13, 5, 11 ____________________

d) 24, 25, 7 ____________________

8. Michel says, "If you multiply each side of a 3–4–5 triangle by the same number, you get another right triangle." Do you agree? Give examples to support your answer.

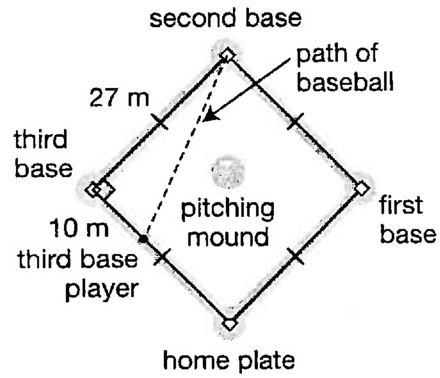
8.2 Applying the Pythagorean Theorem

Try These

Is each set of numbers a Pythagorean triple?

i) 1.2, 0.5, and 1.3 **ii)** 18.2, 3.1, and 2.3

How far must the third base player throw the ball to get the runner out at second base?



1 Which is the unknown length?

____

2 Use the Pythagorean theorem to calculate the unknown length.

$a^2 + b^2 = c^2$

______ + ______ = ______

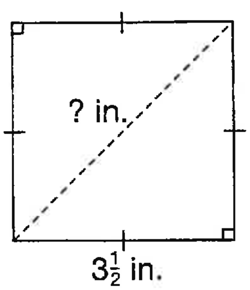
$c^2 =$ ______, so $c = \sqrt{\_\_\_\_\_\_}$, which is _______

3 What is the distance, to one decimal place?

The player must throw the ball ______ m.

Example 1

Beth, a carpenter, needs to estimate the length of the diagonal across a wooden post. How can she estimate without measuring?



Solution

A. Which is the unknown length? ______________

B. What is the square of a leg, $\frac{7}{2}$ in.? $\left(\frac{7}{2}\right)^2 =$ _______

C. What is the sum of the squares of the legs?

Hint
Round to numbers that you can easily calculate the square root of.

D. Estimate the square of the sum. ________

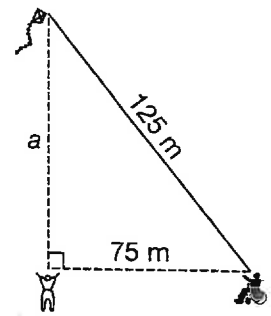
E. What is the estimated length of the hypotenuse?

$c^2 \doteq$ ________, so $c \doteq \sqrt{\rule{2cm}{0.4pt}}$, which is ________.

The diagonal is about ________ long.

Example 2

Jess's grandfather let out 125 m of kite string and is holding the reel 1 m above the ground. Jess is standing 75 m away, directly underneath the kite. How far above the ground is the kite?



Solution

A. Which is the unknown length? ______________

B. How can you calculate the unknown length?

The square of the hypotenuse is ________^{2} = ________.

The square of the known leg is ________^{2} = ________.

Set up the equation and solve it.

a^2 + ________ = ________

a^2 = ________ − ________

a^2 = ________, so $a = \sqrt{\rule{2cm}{0.4pt}}$, or ________

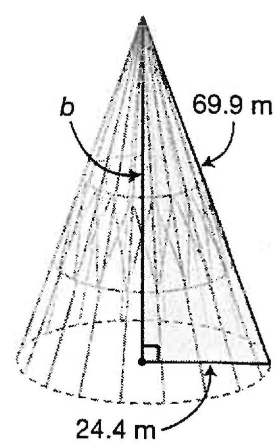
C. How far above the ground is the kite?

REFLECTING

How do you rearrange the formula $c^2 = a^2 + b^2$ to determine the length of a leg?

Example 3

The Saamis Teepee in Medicine Hat, Alberta, is the world's tallest teepee. Each pole is 69.9 m long and touches the ground 24.4 m from the centre of the base. What is the height of the teepee?



Solution

Set up the equation and solve it.

______^{2} + b^2 = ______^{2}

b^2 = ______^{2} − ______^{2}

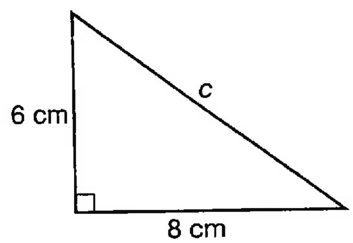
b^2 = ________, so $b = \sqrt{\rule{2cm}{0.4pt}}$, or ________

The height of the teepee is about ______ m.

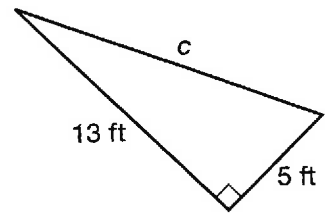
Practice

1. What is the length of the hypotenuse? If necessary, round to the nearest whole unit.

a)

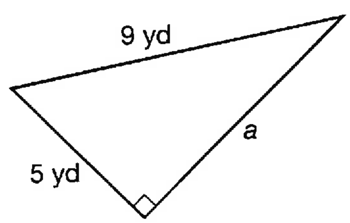


b)



2. What is the length of the unknown leg? If necessary, round to one decimal place.

a)



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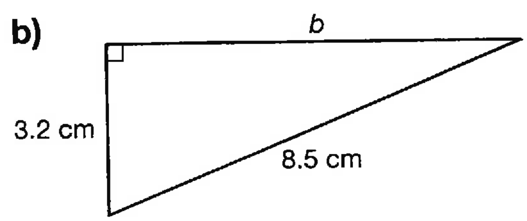
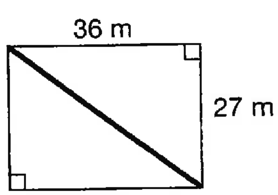
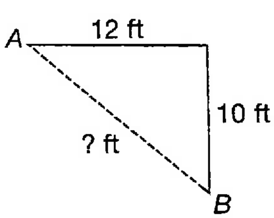


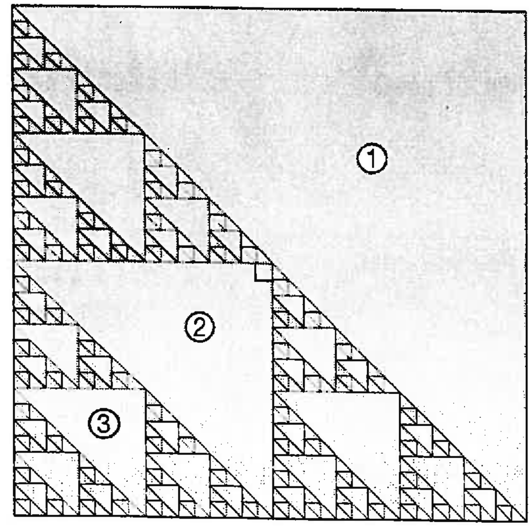


3. A path is being constructed between the corners of a park. What is the length of the path?



4. Luc is building a shed. What should the measure of the diagonal be so that the 12 ft wall is perpendicular to the 10 ft wall?

5. Salwa, a graphic artist, learned about the Sierpinski triangle. She is making a poster of it where the square covers 1 sq ft.



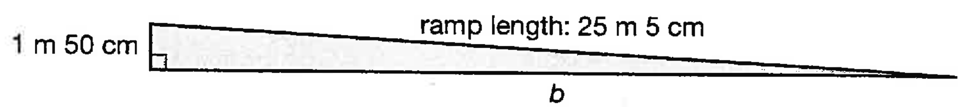
a) Salwa said, "In triangle ①, I can use 12 in. for each of the two shorter sides and 17 in. for the longest side." Will this work? Explain your thinking.

b) In triangle ②, each leg is 6 in. What is the length of the hypotenuse?

REFLECTING

How can you predict the side lengths of the next triangle in the pattern?

6. A wheelchair ramp must be at least 16 m along the ground (the run) for every metre of height (the rise).



a) What is the required ratio of rise to run, expressed as a decimal?

b) Does this ramp meet the specifications? Show your thinking.

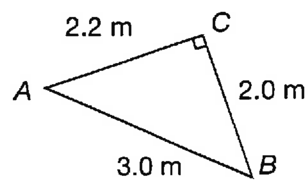
7. The size of a flat screen TV is given by the length of the diagonal of the screen. What is the size of this TV?

8.3 Calculating Length: Sine Ratio

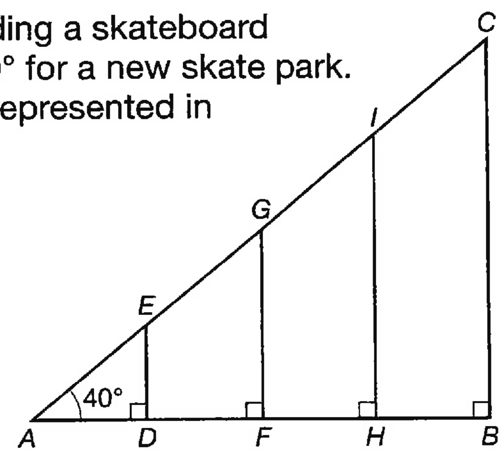
You will need
- a protractor
- a ruler

Try These

i) Put a circle around the label on the **opposite** side of $\angle A$.
ii) Put a triangle around the label on the **hypotenuse.**
iii) Put a box around the label on the **adjacent** side of $\angle A$.



The City of Airdrie, Alberta, is building a skateboard ramp with an elevation angle of 40° for a new skate park. How long are the support beams represented in this diagram?



1 What sides are opposite $\angle A$? Name two.

____

2 Measure the lengths as accurately as you can. Record your results in the chart.

Hint
You will be using this chart again in Lessons 8.4 and 8.5.

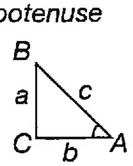
Hypotenuse	Opposite side of $\angle A$	Adjacent side of $\angle A$
AE = _____ mm	ED = _____ mm	AD = _____ mm
AG = _____ mm	GF = _____ mm	AF = _____ mm
_____ = _____ mm	_____ = _____ mm	_____ = _____ mm

3 What is the ratio for $\dfrac{\text{opposite side of } 40°}{\text{hypotenuse}}$?

$\dfrac{ED}{AE}$ = __________ $\dfrac{GF}{AG}$ = __________

What do you notice? ________________________

sine
the ratio of the length of the *opposite* leg to the length of the *hypotenuse*



$\sin A = \frac{a}{c}$

You can use a scientific calculator to determine the **sine** of an acute angle in a right triangle.

4 What is the sine of 40°? $\sin 40°$ = ________

What do you notice? ________________________

5 Draw two different right triangles with a 25° angle. Measure the sides. Compare $\frac{\text{opposite side of } 25°}{\text{hypotenuse}}$ in the two triangles. What do you notice?

____

6 What conclusion can you make, based on your results?

____

____

Tech Tip

Sine

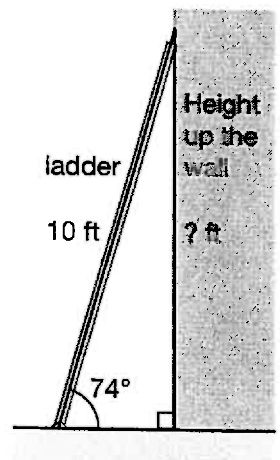
Make sure you are in Degree mode. Try calculating sin 40° by pressing sin 40 = or 40 sin.

Hint

Save your diagrams for later use in this chapter.

Example 1

Lois is cleaning the rain gutters around a roof. At what height does her ladder touch the building? (Round to one decimal place.)



Solution

$$\sin 74° = \frac{\text{opposite side of } 74°}{\text{hypotenuse}}$$

$$\sin 74° = \frac{?\text{ ft}}{\boxed{}\text{ ft}}$$

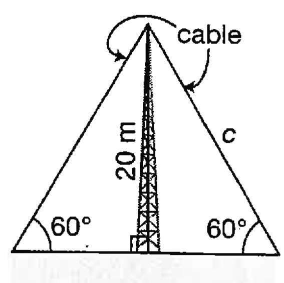
$$\sin 74° \times 10\text{ ft} = \frac{?\text{ ft}}{\boxed{}\text{ ft}} \times \_\_\_\_\_\_\text{ ft}$$

______ × ______ ft = ? ft   ? = ______ ft

The height up the wall is about ______ ft.

Example 2

A communications tower is 20 m tall. For extra support, cables must be attached. How much cable is needed, to the nearest metre?



Solution

$$\sin 60° = \frac{\text{opposite side of } 60°}{\text{hypotenuse}}$$

$$\sin 60° = \frac{\boxed{}\text{ m}}{c}$$

$$c \times \sin 60° = \_\_\_\_\_\_\text{ m}$$

$$c = \frac{\boxed{}\text{ m}}{\sin \boxed{}}, \text{ so}$$

$$c = \_\_\_\_\_\_\text{ m}$$

About ______ m of cable is needed.

Practice

REFLECTING
If the length of the opposite side is equal to the sine value, what is the length of the hypotenuse?

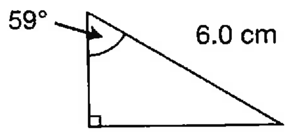
1. Calculate the sine value for each angle.

   a) sin 33° = ____________ **c)** sin 50° = ____________

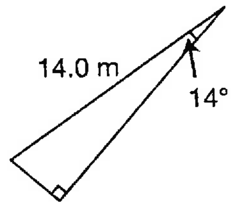
   b) sin 3° = ____________ **d)** sin 68° = ____________

2. Calculate, then label the length of the side that is opposite the given angle. Express your answer to the nearest hundredth.

   a)

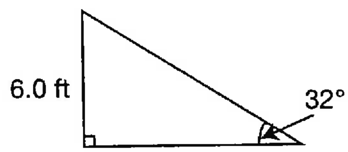


   b)

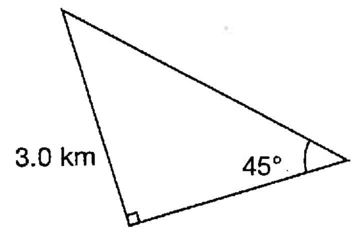


3. Calculate, then label the length of the hypotenuse. Express your answer to the nearest hundredth.

   a)

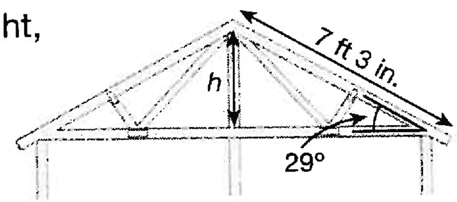


   b)



Hint
Express the length of the hypotenuse as a decimal.

4. In this roof truss, what is the height, *h*, to the nearest tenth?



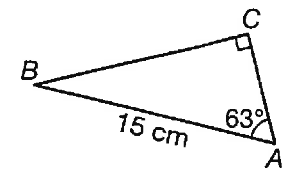
5. How long is the wheelchair ramp, to the nearest centimetre?

Mid-Chapter Review

1. Calculate each side length in ΔABC, to one decimal place.

   a) side BC

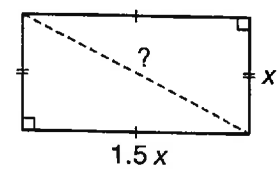
   b) side AC



2. The length of a rectangular box must be 1.5 times its width.

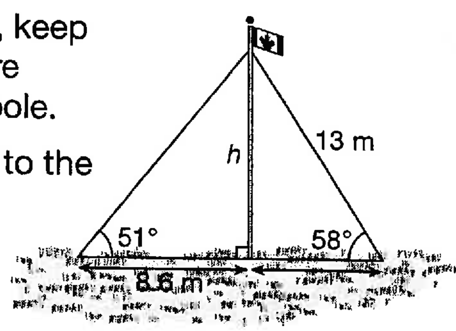
   a) What two other sets of dimensions are possible?
   6 ft by 4 ft, or ________________ or ________________

   b) Samuel is putting a divider on the diagonal to make two spaces in a 6 ft by 4 ft box. He said the diagonal should be $7\frac{1}{2}$ ft long. Will this fit? Explain.



3. Two guy wires, of unequal lengths, keep this flagpole vertical. Both wires are attached 3 m from the top of the pole.

   a) What is the height of the pole, to the nearest metre?



Hint
For Part a), use the triangle at the right.

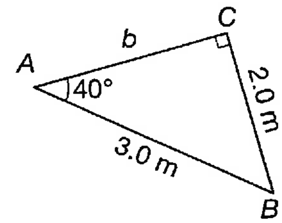
   b) The other wire is attached on the ground, 8.6 m from the pole. What is the length of this wire? (Show two solutions.)

8.4 Calculating Length: Cosine Ratio

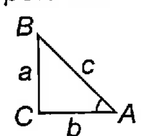
You will need
- a ruler

Try These

i) What is the measure of $\angle B$? ________
ii) Calculate b to one decimal place.

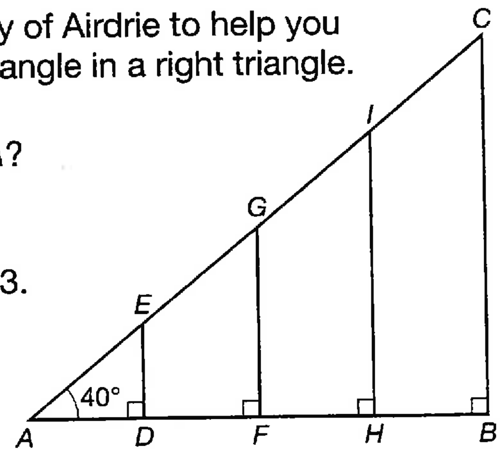


cosine
the ratio of the length of the *adjacent* leg to the length of the *hypotenuse*



$\cos A = \frac{b}{c}$

Use the skateboard ramp of the City of Airdrie to help you understand the **cosine** of an acute angle in a right triangle.



1. What sides are adjacent to $\angle A$? Name two. ________________

2. Use your chart from Lesson 8.3. What is the ratio for $\frac{\text{adjacent side of } 40°}{\text{hypotenuse}}$?

   $\frac{AD}{AE} = \frac{\_\_\_\_\_\_}{\_\_\_\_\_\_}$ or ________

   $\frac{AF}{AG} = \frac{\_\_\_\_\_\_}{\_\_\_\_\_\_}$ or ________

   What do you notice? ________________________________

Tech Tip

Cosine
Try calculating cos 40° by pressing cos 40 = or 40 cos.

3. What is the cosine of 40°? cos 40° = ________
   What do you notice? ________________________________

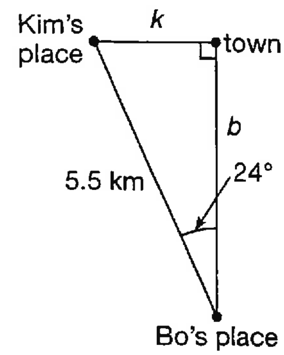
Hint
Use the triangles you drew in Lesson 8.3.

4. Draw two different right triangles with a 25° angle. Measure the sides. Compare $\frac{\text{adjacent side of } 25°}{\text{hypotenuse}}$ in the two triangles.
   What do you notice? ________________________________

5. What conclusion can you make, based on your results?
   ____
   ____

REFLECTING
How can you confirm your conclusion, using different acute angles in a right triangle? Try it.

Example



During winter, Kim and Bo snowmobile to each other's place. How far is it from each friend's place to the nearest town, to the nearest tenth of a kilometre?

Solution

A. What is given? angle: ______; hypotenuse: ______ km

B. How far is town from Bo's place? from Kim's place?

$\cos \underline{\quad\quad} = \dfrac{\boxed{}\text{ km}}{\boxed{}\text{ km}}$

______ × cos ______ = b

________ = b

Bo's place is about ______ km from town.

$\cos \underline{\quad\quad} = \dfrac{\boxed{}\text{ km}}{\boxed{}\text{ km}}$

______ × cos ______ = k

________ = k

Kim's place is about ______ km from town.

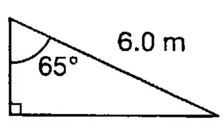
Practice

1. Calculate the cosine value for each angle.

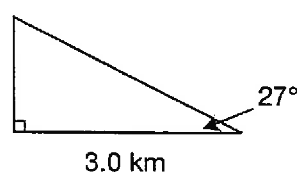
   a) cos 14° = ________ **b)** cos 50° = ________

2. Calculate, then label the lengths of the unknown sides. Express your answer to the nearest tenth.

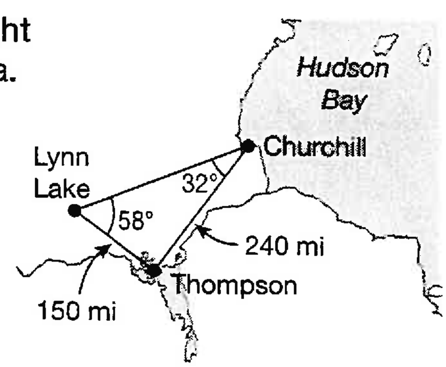
   a)



   b)



3. A bush pilot used this diagram to calculate the straight flying distance from Churchill to Lynn Lake, Manitoba. Show two possible solutions.

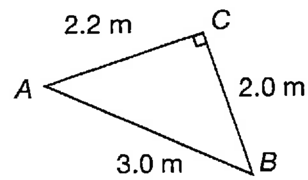


8.5 Calculating Length: Tangent Ratio

You will need
- a ruler

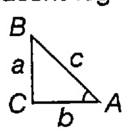
Try These

Mark an arc for $\angle A$. Write the ratio comparing the length of the opposite side to the length of the adjacent side for $\angle A$.



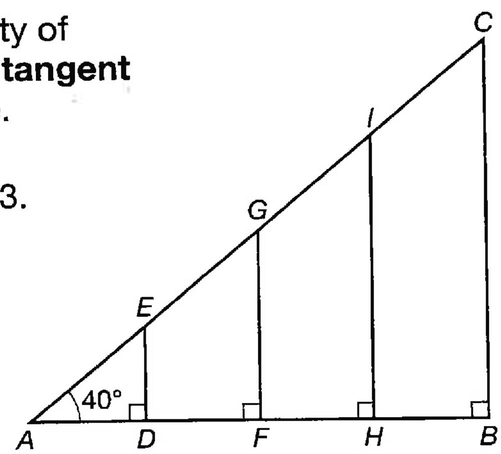
____

tangent
the ratio of the length of the *opposite* leg to the length of the *adjacent* leg



$\tan A = \frac{a}{b}$

Use the skateboard ramp of the City of Airdrie to help you understand the **tangent** of an acute angle in a right triangle.



1. Use your chart from Lesson 8.3.
What is the ratio for $\frac{\text{opposite side of } 40°}{\text{adjacent side of } 40°}$?
$\frac{ED}{AD} = \frac{\quad}{\quad}$ or ________
$\frac{GF}{AF} = \frac{\quad}{\quad}$ or ________
What do you notice? ____________________

Tech Tip

Tangent
Try calculating tan 40° by pressing tan 40 = or 40 tan.

2. What is the tangent of 40°? tan 40° = ________
What do you notice? ____________________

Hint
Use the triangles you drew in Lesson 8.3.

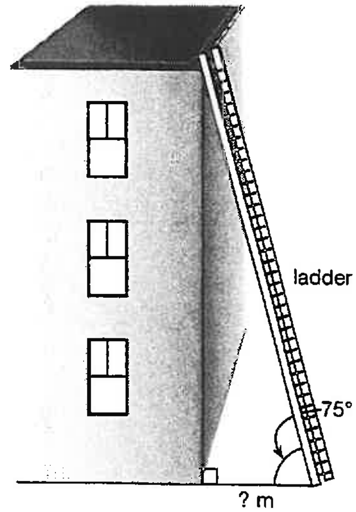
3. Draw two different right triangles with a 25° angle. Measure the sides. Compare $\frac{\text{opposite side of } 25°}{\text{adjacent side of } 25°}$ in the triangles.
What do you notice? ____________________

REFLECTING
How can you confirm your conclusion using different right triangles? Try it.

4. What conclusion can you make, based on your results?
____
____

Example

In an emergency, a firefighter estimates the height of a building: 3 storeys × 4 m per storey + 1 m = 13 m. How far away from this building should the ladder be placed?



Solution

$$\tan \_\_\_\_\_\_ = \frac{\square \text{ m}}{\square \text{ m}}$$

$$? \times \tan \_\_\_\_\_\_ = \_\_\_\_\_\_ \text{ m}$$

$$? = \frac{\square \text{ m}}{\tan \square}, \text{ so } ? = \_\_\_\_\_\_ \text{ m}$$

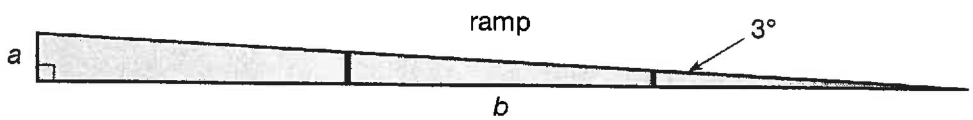
The ladder should be placed about ______ m from the building.

REFLECTING

How can you check your answer?

Practice

1. A wheelchair ramp is built like a skateboard ramp.



   a) What is the tangent of 3°? tan 3° = ________

   b) Write three ratios that are approximately equal to tan 3°.

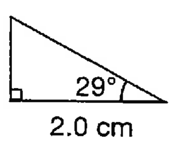
   $\frac{\text{opposite}}{\text{adjacent}} =$ ________, ________, ________

Hint

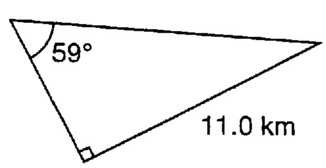
You can include units for the lengths in the ramp.

2. Calculate, then label, the lengths of the legs. Express your answer to the nearest hundredth.

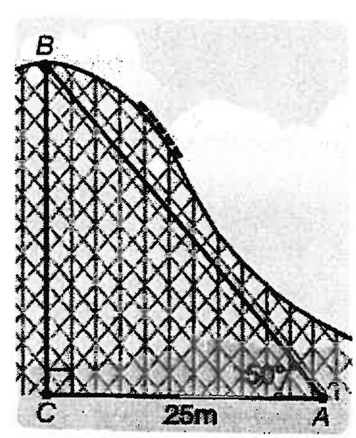
   a)



   b)



3. Tabitha counts off 25 paces from the base of a roller coaster. From this position, she estimates that *CA* is about 25 m, and the elevation angle, $\angle BAC$, is 50°. What is the greatest height on this roller coaster, to one decimal place?



8.6 Trigonometry: Calculating Angles

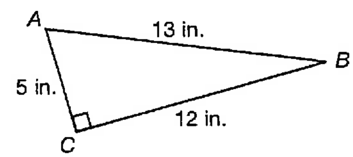
Try These

Is the given ratio for $\angle A$ the trig ratio for sine, cosine, or tangent?

i) $\frac{5}{13}$ = ________ of $\angle A$

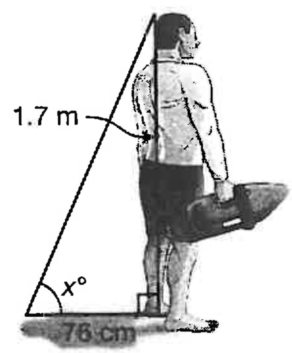
ii) $\frac{12}{5}$ = ________ of $\angle A$

iii) $\frac{12}{13}$ = ________ of $\angle A$



angle of elevation
the angle between the horizontal and the line of sight when looking *up* at an object

Gyrolf is a lifeguard. At 2 o'clock, his shadow is 76 cm. What is the **angle of elevation** of the sun at that time, to the nearest degree?



REFLECTING
Why must the measurements have the same unit?

1. What trig ratio relates the unknown angle, $x°$, to the side lengths you know?

   $\frac{\text{opposite side of } x°}{\text{adjacent side of } x°}$ is the ________ of $x°$.

2. What is the equation? $\tan x° = \frac{170}{\square}$, or $\tan x° = \frac{1.7}{\square}$

Tech Tip
If $\tan x° = \frac{3}{4}$, try calculating the value of x by pressing 2nd tan (3 ÷ 4) = or 3 ÷ 4 = 2nd tan. Your calculator should show 36.8698..., which is about 37°.

To determine the value of $x°$, you need to do the *inverse* operation on your calculator. You calculate the inverse of a trig ratio by pressing the 2nd function key. The inverse of tan is $\tan^{-1}$.

3. Solve for x. $\tan x° = \frac{170}{\square}$, so

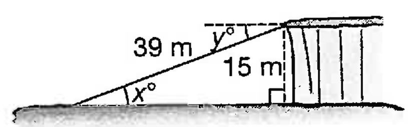
   $x° = \tan^{-1}\left(\frac{170}{\square}\right)$, which is ________

   The angle of elevation of the sun is about ________.

angle of depression
the angle between the horizontal and the line of sight when looking *down* at an object

Example

A cable is secured at the top of a cliff to create a zip line.



- What angle does the zip line make with the ground?
- What is the **angle of depression**, from the top of the cliff?

Solution

A. What angle does the zip line make with the ground?

$\dfrac{\text{opposite side of } x^\circ}{\text{hypotenuse}}$ is the ______ of x°. $\sin x^\circ = \dfrac{\square \text{ m}}{\square \text{ m}}$, so

$x^\circ = \sin^{-1}\left(\dfrac{\quad}{\quad}\right)$

x° = ________, which is about ______

B. How can the horizontal lines help you relate angles x and y?

____

____

C. What is the angle of depression? y = ______

> **Tech Tip**
>
> If $\sin x^\circ = \frac{1}{2}$, try calculating the value of x by pressing 2nd sin (1 ÷ 2) = or 1 ÷ 2 = 2nd sin. You should get 30, which means 30°.

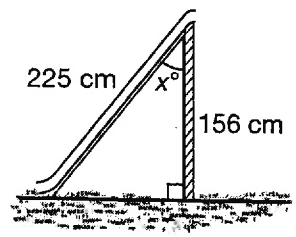
Practice

1. a) At what angle does the slide meet the ladder?

$x^\circ = \cos^{-1}($______$)$, so $x \doteq$ ______

b) What is the angle of depression from the top of the slide?

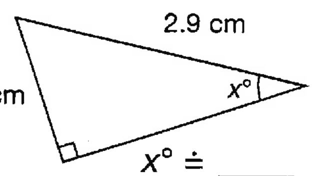
____



2. Calculate the measure of the angle, to the nearest degree.

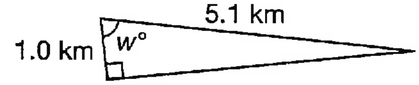
> **Hint**
>
> First, determine what trig ratio you will use.

a)



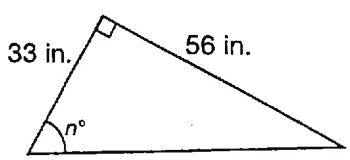
$x^\circ \doteq$ ______

c)



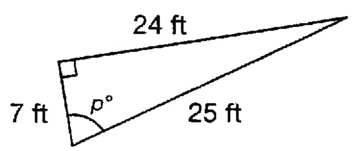
$w^\circ \doteq$ ______

b)



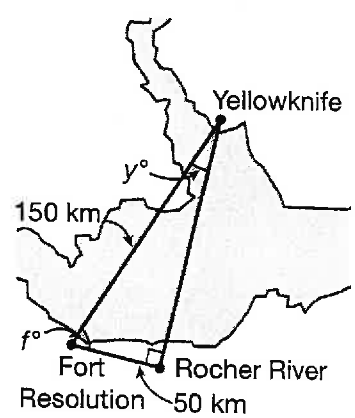
$n^\circ \doteq$ ______

d)



$p^\circ \doteq$ ______

3. A trader flies between the locations shown. At what angle does the pilot turn to reach each place? Round to one decimal place.

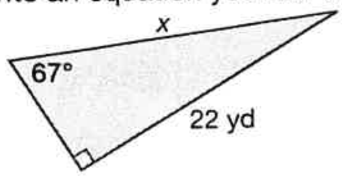


8.7 Solving Right Triangle Problems

Try These

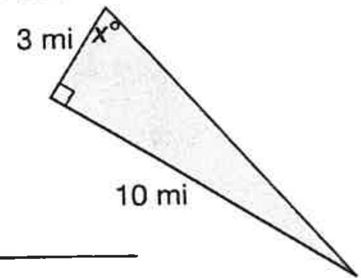
Write an equation you can use to find the value of x.

i)



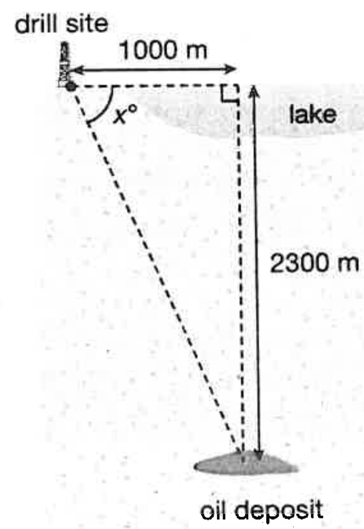
$x =$ ____________

ii)



$x° =$ ____________

Jamie works for an oil company. He needs to drill a well to an oil deposit below the surface of a lake. The drill site is located on land as shown. What is the angle of depression for drilling the well? (Round to the nearest tenth.)



1 What equation can you use to calculate $x°$?

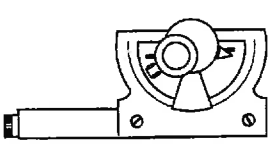
$\tan x° = \frac{\boxed{}}{\boxed{}}$, so

$x° =$ ____________

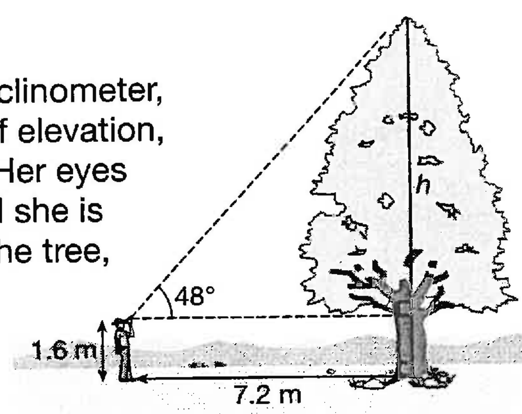
2 Calculate the angle to the nearest tenth. $x° =$ ____________

The well should be drilled at an angle of ____________.

Example



Glenda is a forester. She uses a clinometer, a device that measures angles of elevation, to sight the top of a tree at 48°. Her eyes are 1.6 m above the ground, and she is 7.2 m from the tree. How tall is the tree, to one decimal place?



Solution

A. What equation can you use to calculate the length of the side opposite the 48° angle?

$$\tan \_\_\_\_\_ = \frac{h}{\square}$$

B. Solve it. $\tan \_\_\_\_\_ \times \_\_\_\_\_ = h$

$$\_\_\_\_\_\_ = h$$

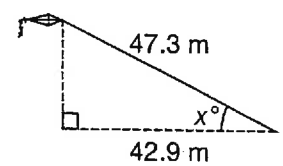
The height of the opposite side is _______ m, to the nearest tenth.

REFLECTING
How can you check your answers?

C. How tall is the tree? _______ m + _______ m = _______ m

Practice

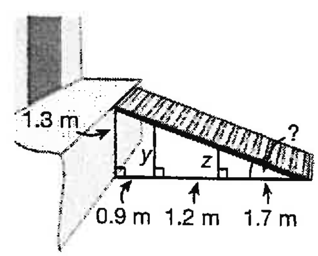
1. What is the angle of elevation of the kite to the nearest degree?



$x° = \cos^{-1}\left(\frac{\square}{\square}\right)$, so $x° =$ _________

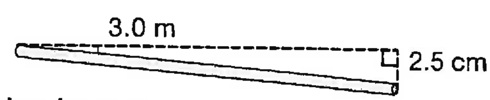
____

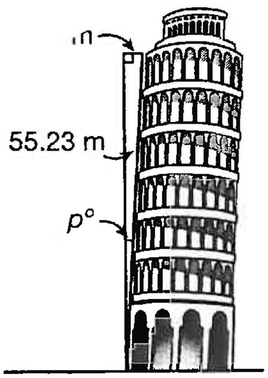
2. a) What angle does the loading ramp make with the ground, to the nearest degree?



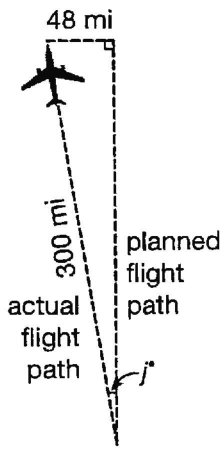
b) What are the lengths of y and z, the support braces, to one decimal place?

3. A plumber is laying drainage pipe for a septic system. For every 3.0 m of horizontal distance, there should be a 2.5 cm drop in height for the water to run. At what angle to the surface must she lay the pipe?



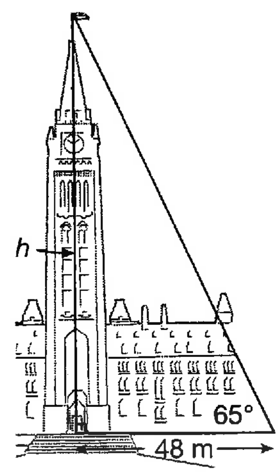


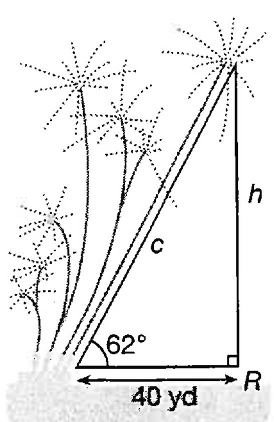
4. The Leaning Tower of Pisa, in Italy, leans because the ground underneath it is unstable. When these measurements were taken, what was p, the angle of the tilt of the tower?



5. After an hour of flying, a jet has travelled 300 miles, but strong winds have blown the jet off course. The instruments in the cockpit show that the jet is 48 miles west of the planned flight path. By how many degrees is the jet off course? (Round to one decimal place.)

6. Patricia is at the Peace Tower in Ottawa. Using a theodolite, she measures a 65° angle to the top of the Canadian flag. If the device is 1.5 m from the ground, what is the height, h, of the flag? (Round to one decimal place.)





7. Roxanne is sitting on the ground, at R, watching fireworks. The diagram shows one of the fireworks exploding directly above her head.

a) At what height above ground does it explode, to the nearest yard?

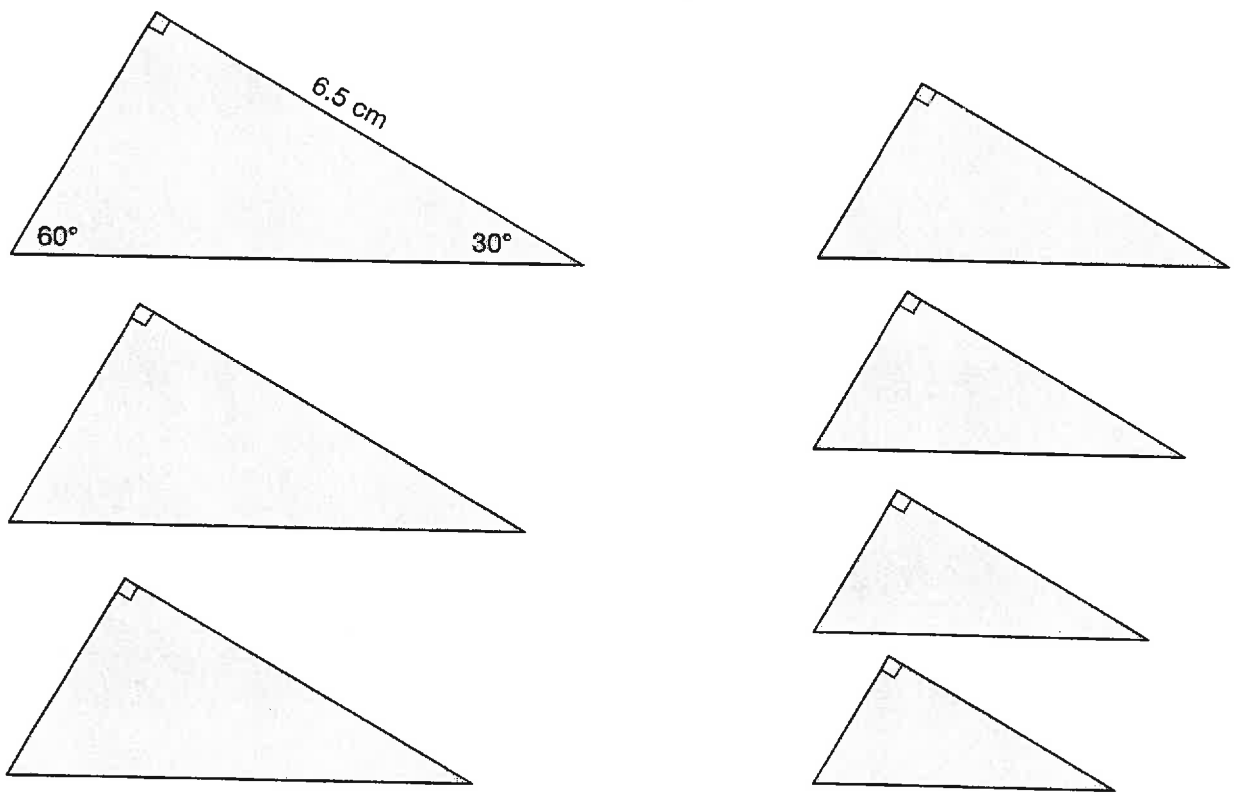
b) About how many yards does it travel before exploding?

Solving a Triangle Puzzle

8.8

Each of the seven triangles below is a 30–60–90 right triangle.

You will need
- scissors



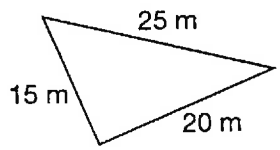
A. What is the length of the longest side, to one decimal place?

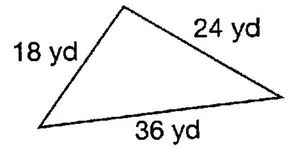
B. Trace the triangles and cut them out. Use all seven triangles to form a single square, with no overlapping.

C. Describe the strategy you used to form the square.

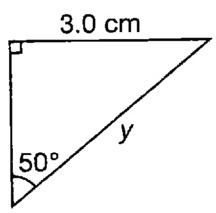
Chapter Review

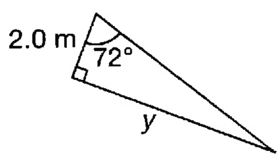
1. Determine if each triangle below is a right triangle or not.

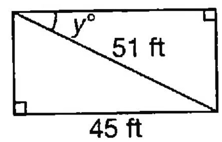
   a) 

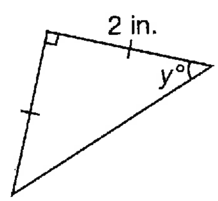
   b) 

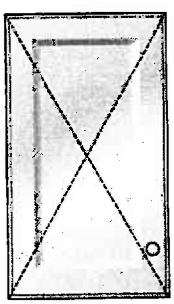
2. Calculate the value of y in each right triangle.

   a) 

   b) 

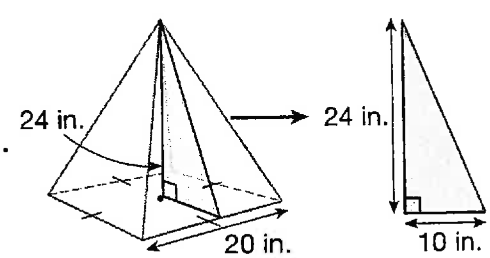
   c) 

   d) 



3. Yannick is creating rectangular doors for his kitchen cabinets. Each door is 20 in. by 36 in. What is the measure of the diagonals?

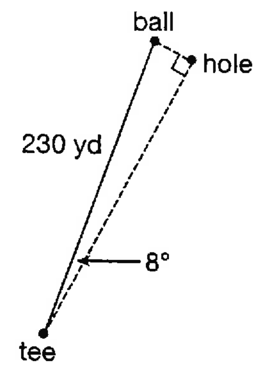
4. Dorothy needs to know the slant height before she can calculate the area of a pyramid. Calculate the slant height.



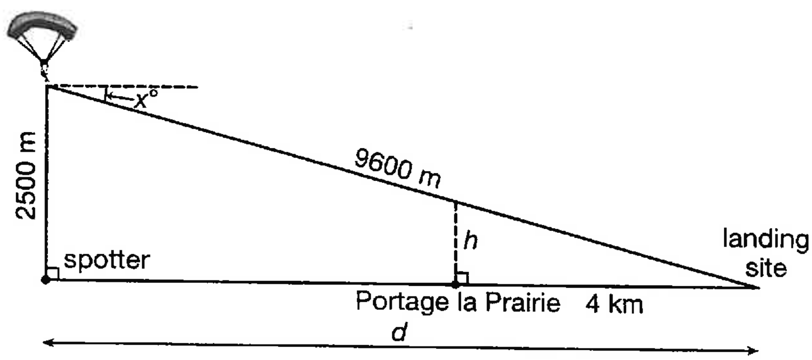
5. Arthur, a golfer, hit his golf ball 230 yd from the tee.

   a) How far is the hole from the tee, to the nearest yard?

   b) How many yards is the ball from the hole?



6. Lucie is a parachute jumper. She jumped from the plane when she was directly over her spotter on the ground.

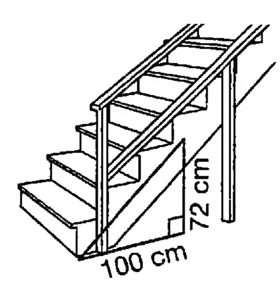


   a) Calculate the distance, d, that her spotter must travel to the place Lucie will land. (Round to the nearest 10 m.)

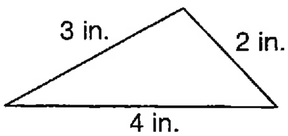
   b) What is the angle of depression to her landing site, when she jumps from the plane? (Round to the nearest degree.)

   c) During this jump, she passes over the town of Portage la Prairie, Manitoba. What is her altitude, h, when she passes over the town?

7. A building code states that for stairs, the steepest angle is 72 cm of rise for each 100 cm of run. What is the steepest angle for building stairs, to the nearest degree?



Chapter Test



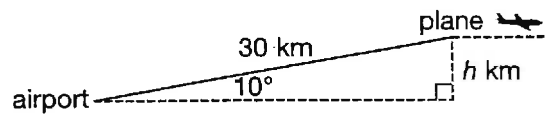
1. Explain how to use the Pythagorean theorem to determine whether this is a right triangle.

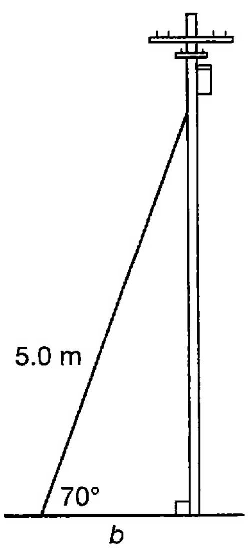
___

___

___

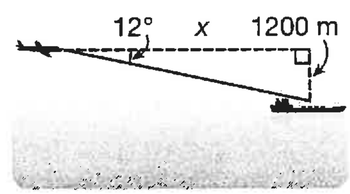
2. During takeoff, a plane forms a 10° angle with the ground. At what height does the plane level off?

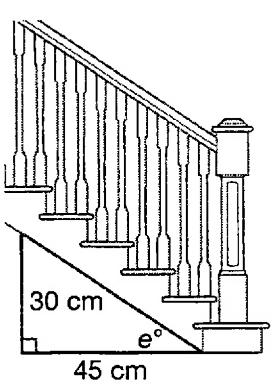




3. How far from the base of this pole can the 5.0 m wire be staked?

4. A plane is flying to a ship in distress. The pilot estimates the angle of depression as 12°. How many more kilometres must the plane travel until it is directly above the ship?





5. What is the angle of elevation of the railing, to the nearest degree?

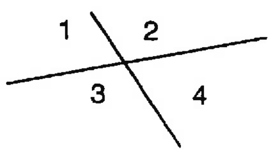
Glossary

A

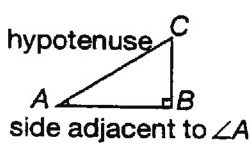
acres: a unit of measure for area in the imperial system
1 acre = 4840 sq yd
1 acre ≐ 0.405 ha

acute angle: an angle that measures more than 0° and less than 90°

adjacent angles: angles that share a common vertex and a common arm
For example, angles 1 and 2 are adjacent angles. Angles 3 and 4 are adjacent angles.

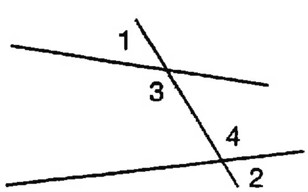


adjacent side: the side that is part of an **acute angle** in a **right triangle** but is not the **hypotenuse**
For example, AB is adjacent to $\angle A$.



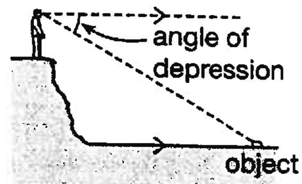
adjacent sides: two sides in a triangle or **polygon** that share a vertex

alternate angles: two angles formed by two lines and a **transversal** and located on opposite sides of the transversal
For example, angles 3 and 4 are alternate interior angles. Angles 1 and 2 are alternate exterior angles.

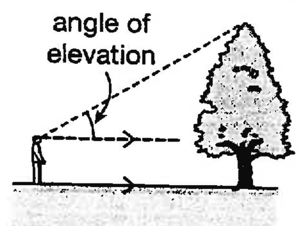


angle bisector: a line that cuts an angle in half to form two equal angles

angle of depression: the angle between the horizontal and the line of sight when looking down at an object



angle of elevation: the angle between the horizontal and the line of sight when looking up at an object



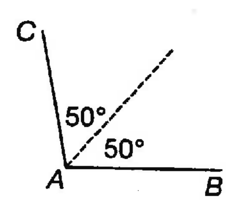
annual: for a year

B

base salary: payment for a given work period, such as an hour or a week, but not including additional pay

bisect: to divide into two equal parts

bisector: the line that divides an angle or line into two equal parts



bonus: an additional payment to a worker as a reward for meeting company goals

C

Canada Pension Plan (CPP): a government fund that provides a monthly pension to workers when they retire

capacity: the amount that a container can hold

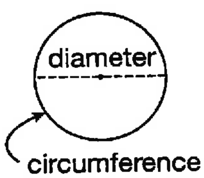
Celsius: a scale for temperature that includes the freezing point of water at 0° and the boiling point of water at 100°

centimetre (cm): a unit of measure for length in the metric system
1 cm = 10 mm
100 cm = 1 m

centre of rotation: a fixed point around which points in a shape are rotated. It can be inside or outside the shape.

charitable donations: an option for employees to make a regular donation to a charity

circumference: the perimeter of a circle
Circumference = $\pi \times d$, where d is the **diameter**
(π is about 3.14)



commission: a payment based on a percentage of the worker's sales

company health plan: a plan for medical expenses not covered by other government health care plans

company pension plan: a fund that provides a company pension during retirement, in addition to **CPP**

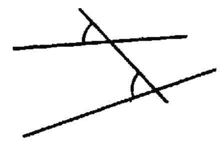
complementary angles: two angles whose sum is 90°

congruent: same size and shape

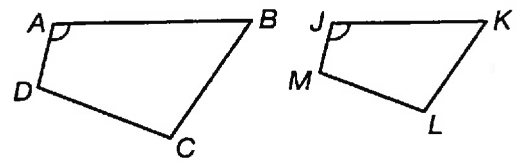
contract: a payment for a fixed period of time and/or a fixed amount of money

coordinates (*x*, *y*): a way to describe locations on a grid using a pair of numbers
For example, (−1, 3) lines up with −1 on the *x*-axis and 3 on the *y*-axis.

corresponding angles: **1.** two angles formed by two lines and a transversal and located on the same side of the transversal

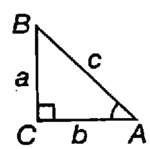


2. angles that match when two shapes are arranged to look the same



corresponding sides: sides that match when two shapes are arranged to look the same
For example, AB and JK are corresponding sides (above).

cosine: the ratio of the length of the **adjacent** leg to the length of the **hypotenuse** in a **right triangle**



$\cos A = \frac{b}{c}$

cup (c): a unit of measure for **capacity** in the imperial system
1 cup = 8 fluid ounces (US) or 10 fluid ounces (UK)
2 cups = 1 pint

D

decametre (dam): a unit of measure for length in the metric system
1 dam = 10 m
100 dam = 1 km

decimetre (dm): a unit of measure for length in the metric system
1 dm = 10 cm
10 dm = 1 m

diameter: a straight line through the centre of a circle that joins two points on the **circumference**
Diameter = **radius** × 2

dilation: the result of multiplying or dividing each length on a shape by the same number to create a similar shape

dilation centre: a fixed point from which a shape is enlarged or reduced

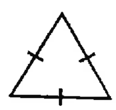
disability insurance: a plan that provides a source of income when an employee is injured and unable to work

double time: the hourly wage multiplied by 2

E

Employment Insurance (EI): a fund that provides income to people who lose their jobs (through no fault of their own) while they look for a new job

equilateral triangle: an equilateral triangle has equal sides and equal angles



F

face: a 2-D shape that forms a flat surface of a 3-D object

Fahrenheit: a scale for temperature that includes the freezing point of water at 32° and the boiling point of water at 212°

fluid ounce (fl oz): a unit of measure for **capacity** in the imperial system
1 fluid ounce = 2 tablespoons
8 fluid ounces = 1 cup (US) or
10 fluid ounces = 1 cup (UK)

foot (ft): an imperial unit of measurement for length
1 foot = 12 inches
3 feet = 1 yard

G

gallon (gal): a unit of measure for **capacity** in the imperial system
1 gallon = 4 quarts

gram (g): a unit of measure for **mass** in the metric system
1000 g = 1 kg

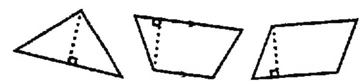
gross income: the total amount of money earned in a pay period before any deductions

H

hectares (ha): a unit of measure for area in the metric system
1 ha is the same area as 1 square **hectometre**
1 ha = 1 hm^2

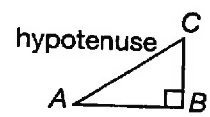
hectometre (hm): a unit of linear measure in the metric system
1 hm = 100 m
10 hm = 1 km

height: the perpendicular distance from the base of a **polygon** to an opposite vertex



hourly wage: a fixed payment for each hour of work

hypotenuse: the side of a **right triangle** that is opposite the 90° angle



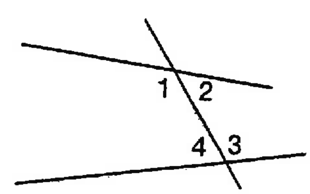
I

inch: an imperial unit of measurement for length
12 inches = 1 foot
36 inches = 1 yard

income: money received for work

income tax: a portion of a worker's earnings that federal and provincial governments use to provide services

interior angles: **1.** angles inside a polygon
2. angles between two lines
For example,



irregular polygon: a closed figure with straight sides with varying side lengths and angle measures

K

kilogram (kg): a metric unit of measure for **mass**
1 kg = 1000 g
1000 kg = 1 tonne (t)

kilolitre (kL): a unit of measure for **capacity** in the metric system
1 kL = 1000 L

kilometre (km): a unit of measure for length in the metric system
1 km = 1000 m

L

legs: the two sides that form the 90° angle in a **right triangle** (see **hypotenuse**)

life insurance: a plan that pays a sum of money to a family member or designated beneficiary in the case of an employee's death

line of reflection: the line across which a shape is flipped

litre (L): a metric unit of measure for **capacity**
1 L = 1000 mL
1000 L = 1 kL

M

mass: the amount of matter in an object. Common units of mass are **grams**, **kilograms**, and **tonnes** (metric) and **pounds** and **tons** (imperial).

metre (m): the base unit of measure for length in the metric system
1 m = 100 cm
1000 m = 1 km

midpoint: the point on a line segment that divides it into two equal parts

mile (mi): an imperial unit of measure for length
1760 yards = 1 mile
5280 feet = 1 mile

millilitre (mL): a metric unit of measure for **capacity**
1000 mL = 1 L

millimetre (mm): a unit of measure for length in the metric system
1000 mm = 1 m
10 mm = 1 cm

N

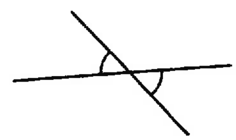
net: a composite 2-D shape that can be folded to create a 3-D object (such as a cube, cone, pyramid, cylinder)

net income: the money left after deductions are taken from gross income; also called take-home pay

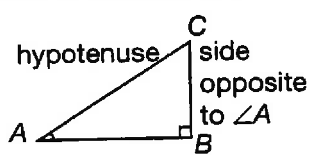
O

obtuse angle: an obtuse angle is greater than 90° but less than 180°

opposite angles: non-adjacent angles that are formed by two intersecting lines



opposite side: the side that is directly across from a specific **acute angle** in a **right triangle**
For example, *BC* is opposite ∠*A*.



ounce (oz): a unit of measure for **mass** in the imperial system
16 ounces = 1 pound

P

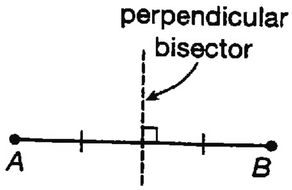
parallel: two or more lines that are always the same distance apart

payroll savings: an option for employees to make a regular contribution to a savings plan, such as Canada Savings Bonds

perimeter: the distance around an object

perpendicular: two lines that form a right angle (90°)

perpendicular bisector: a line that bisects a line segment and is perpendicular to the line segment



pi (π): the ratio of the **circumference** of a circle to its **diameter**. Its value is about 3.14.

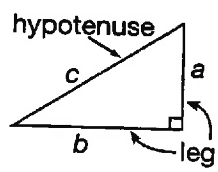
piecework: a payment based on the number of items created or completed

pint (pt): a unit of measure for **capacity** in the imperial system
1 pint = 2 cups
2 pints = 1 quart

polygon: a closed figure with straight sides

pound (lb): a unit of measure for **mass** in the imperial system
1 pound = 16 **ounces**
2000 pounds = 1 **ton**

Pythagorean theorem: a statement of a relationship in which the sum of the squares of the lengths of the **legs** of a **right triangle** is equal to the square of the length of the **hypotenuse**
$a^2 + b^2 = c^2$

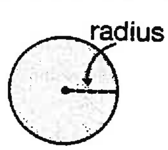


Q

quart (qt): a unit of measure for **capacity** in the imperial system
1 quart = 2 pints
4 quarts = 1 gallon

R

radius: a straight line from the centre of a circle to any point on the **circumference**



rate of exchange: the amount that money is worth from one currency to another. This varies daily.

ratio: a comparison of quantities with the same units

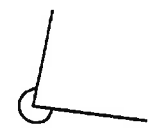
reciprocal: the multiplier of a number that gives 1 as a result
For example, the reciprocal of $\frac{1}{2}$ is $\frac{2}{1}$ or 2.
$\frac{1}{2} \times \frac{2}{1} = 1$ and

$1 \div \frac{1}{2} = \frac{2}{1}$

referent: a known measure used for comparing and estimating

reflection: the result of flipping a 2-D shape across a line

reflex angle: an angle that measures between 180° and 360°



regular polygon: a closed figure with all sides equal and all angles equal

right angle: an angle that measures 90°

right triangle: a triangle that contains a right angle

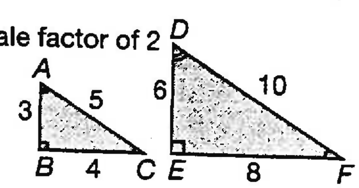
rotation: the result of turning a 2-D shape around a point. Rotations can go clockwise (cw) or counterclockwise (ccw).

royalty: a payment for a piece of work that is marketed and sold. The amount is based on a percentage of sales.

S

salary: a regular fixed payment for work, usually expressed as an amount per year but paid regularly (e.g., every two weeks or monthly)

scale factor: the number that the dimensions of a **polygon** are multiplied by to calculate the corresponding dimensions of a **similar polygon**

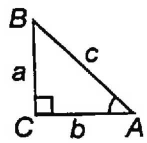


sectors: sections of a circle

shift premium: an additional amount of money for working outside of regular workday hours or on weekends

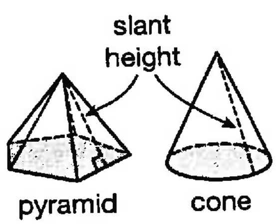
similar polygons: **polygons** that are congruent or are enlargements or reductions of each other. The ratios of corresponding lengths are the same, and corresponding angles are equal.

sine: the ratio of the length of the **opposite** leg to the length of the **hypotenuse** in a **right triangle**



$\text{Sin } A = \frac{a}{c}$

slant height: the distance from the top to the base, at a right angle, along a slanted side of a pyramid or cone. It is measured to the **midpoint** of the base side for a pyramid.



square number: the result when a whole number is multiplied by itself

straight commission: payment based only on sales made

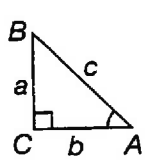
supplementary angles: two angles whose sum is 180°

surface area: the sum of all the areas of the faces of a 3-D object

symmetrical: a way of describing a shape that can be folded along at least one line so one half fits exactly over the other

T

tangent: the ratio of the length of the **opposite** leg to the length of the **adjacent** leg



$\text{Tan } A = \frac{a}{b}$

time and a half: the hourly wage multiplied by a factor of 1.5

ton (T): a unit of measure for **mass** in the imperial system
1 ton = 2000 pounds

tonne (t): a metric unit of measure for **mass**
1 t = 1000 kg

transformation: the result of moving or changing a shape according to a rule. The new shape is called the image.

translation: the result of sliding a 2-D shape along a straight line. On a grid, you can translate a shape right, left, up, or down.

translation rule: a way of describing a translation with numbers and directions
For example, "8 units right and 4 units up" or (R8, U4)

transversal: a line that intersects two or more lines

trigonometry: the study of relationships among the sides and angles in **right triangles**

U

union dues: a deduction made when an employee belongs to a union. Unions negotiate wages, benefits, and working conditions with employers.

unit price: the amount of money charged for a unit of an item

V

vertex: the point where two or more lines meet

volume: the amount of space occupied by a 3-D object

W

wage and tips: an hourly wage plus varying amounts in tips for services provided

Y

yard: an imperial unit of measure for length
1 yard = 3 feet
1 yard = 36 inches

Charts and Formulas

Metric Units

Length	Area	Volume	Capacity	Mass
kilometre (km) 1 km = 1000 m	square kilometre (km^2) 1 km^2 = 1 000 000 m^2 1 km^2 = 100 ha	cubic kilometre (km^3) 1 km^3 = 1 000 000 000 m^3	kilolitre (kL) 1 kL = 1000 L	kilogram (kg) 1 kg = 1000 g 1000 kg = 1 t
hectometre (hm) 1 hm = 100 m	square hectometre (hm^2) 1 hectare (ha) = 1 hm^2 1 ha = 10 000 m^2	cubic hectometre (hm^3) 1 hm^3 = 1 000 000 m^3	hectolitre (hL) 1 hL = 100 L	hectogram (hg) 1 hg = 100 g
decametre (dam) 1 dam = 10 m	square decametre (dam^2) 1 dam^2 = 100 m^2	cubic decametre (dam^3) 1 dam^3 = 1000 m^3	decalitre (daL) 1 daL = 10 L	decagram (dag) 1 dag = 10 g
metre (m) 1 m = 100 cm	square metre (m^2) 1 m^2 = 10 000 cm^2	cubic metre (m^3) 1 m^3 = 1 000 000 cm^3	litre (L) 1 L = 1000 mL	gram (g) 1 g = 1000 mg
decimetre (dm) 1 dm = 0.1 m	square decimetre (dm^2) 1 dm^2 = 0.01 m^2	cubic decimetre (dm^3) 1 dm^3 = 0.001 m^3	decilitre (dL) 1 dL = 0.1 L	decigram (dg) 1 dg = 0.1 g
centimetre (cm) 1 cm = 0.01 m 1 cm = 10 mm	square centimetre (cm^2) 1 cm^2 = 0.0001 m^2	cubic centimetre (cm^3) 1 cm^3 = 0.000 001 m^3 Note: 1 cm^3 holds 1 mL	centilitre (cL) 1 cL = 0.01 L	centigram (cg) 1 cg = 0.01 g
millimetre (mm) 1 mm = 0.001 m	square millimetre (mm^2) 1 mm^2 = 0.000 001 m^2	cubic millimetre (mm^3) 1 mm^3 = 0.000 000 001 m^3	millilitre (mL) 1 mL = 0.001 L	milligram (mg) 1 mg = 0.001 g

Imperial Units

Length	Area	Volume	Capacity	Mass
inch (in. or ")	square inches (sq in.)	cubic inches (cu in.)	tablespoon (T)	ounces (oz)
foot (ft or ') 1 foot = 12 inches	square feet (sq ft) 1 sq ft = 144 sq in.	cubic feet (cu ft) 1 cu ft = 1728 cu in.	fluid ounce (fl oz) 1 fl oz = 2 T	pound (lb) 1 lb = 16 oz
yard (yd) 1 yard = 3 feet	square yard (sq yd) 1 sq yd = 9 sq ft	cubic yard (cu yd) 1 cu yd = 27 cu ft	cup (c) 1 c = 8 fl oz (US) 1 c = 10 fl oz (UK)	ton (T) 1 T = 2000 lb (US) 1 T = 2240 lb (UK)
mile (mi) 1 mile = 1760 yd	square mile (sq mi) 1 sq mi = 3 097 600 sq yd	cubic mile (cu mi)	pint (pt) 1 pt = 2 c	
	1 acre = 4840 sq yd		quart (qt) 1 qt = 2 pt	
			gallon (gal) 1 gal = 4 qt	

Converting Common Imperial Units to Metric (SI)

Linear	Area	Volume	Capacity	Mass
1 in. $\doteq$ 2.54 cm	1 sq in $\doteq$ 6.4516 cm^2	1 cu in. $\doteq$ 16.39 cm^3	1 fl oz $\doteq$ 29.57 mL	1 oz $\doteq$ 28.35 g
1 ft $\doteq$ 0.31 m	1 sq ft $\doteq$ 0.0929 m^2	1 cu ft $\doteq$ 28.32 dm^3	1 pt $\doteq$ 0.47 L, or 470 mL	1 lb $\doteq$ 0.45 kg
1 yd $\doteq$ 0.91 m	1 sq yd $\doteq$ 0.8361 m^2	1 cu yd $\doteq$ 0.76 m^3	1 qt $\doteq$ 0.95 L, or 950 mL	1 T $\doteq$ 0.91 t
1 mi $\doteq$ 1.61 km	1 sq mi $\doteq$ 2.5900 km^2	1 cu mi $\doteq$ 4.17 km^3	1 gal $\doteq$ 3.79 L, or 3790 mL	
	1 acre $\doteq$ 0.4047 ha			

Converting Common Metric (SI) Units to Imperial

Linear	Area	Volume	Capacity	Mass
1 mm $\doteq$ 0.039 in.			1 mL $\doteq$ 0.03 fl oz	
1 cm $\doteq$ 0.39 in.	1 cm^2 $\doteq$ 0.1550 sq in.	1 cm^3 $\doteq$ 0.06 cu in.		
1 m $\doteq$ 1.09 yd	1 m^2 $\doteq$ 10.7639 sq ft	1 m^3 $\doteq$ 1.31 cu yd	1 L $\doteq$ 2.11 pt	1 g $\doteq$ 0.04 oz
1 m $\doteq$ 3.27 ft			1 L $\doteq$ 1.06 qt	1 kg $\doteq$ 2.21 lb
1 km $\doteq$ 0.62 mi	1 km^2 $\doteq$ 0.3861 sq mi	1 km^3 $\doteq$ 0.24 cu mi	1 L $\doteq$ 0.26 gal	1 t $\doteq$ 1.10 T

Temperature

$$F = \frac{9}{5}C + 32$$

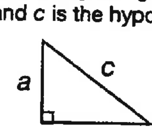
$$C = \frac{5}{9}(F - 32)$$

Circle Formulas

Diameter = radius × 2
Circumference
= π × diameter
Circumference
= π × radius × 2
Area: $\pi \times r^2$

Pythagorean theorem

$a^2 + b^2 = c^2$, where a and b are sides adjacent to the right angle in a right triangle and c is the hypotenuse



Primary Trigonometric Relationships

$$\sin A^\circ = \frac{\text{opposite side of } A^\circ}{\text{hypotenuse}}$$

$$\cos A^\circ = \frac{\text{adjacent side of } A^\circ}{\text{hypotenuse}}$$

$$\tan A^\circ = \frac{\text{opposite side of } A^\circ}{\text{adjacent side of } A^\circ}$$