GRADE 10

7 Lines and Angles

Tina is a carpenter who specializes in wood flooring. When she lays the flooring, she is careful to keep the pieces straight. When she creates a design, she likes to vary the angles.

A. What angles do you think are used most often in carpentry?

B. What tools can be used to measure angles and lines?

7 Getting Started

You will need
- a ruler
- a protractor

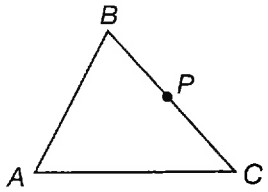
1. Name each of these in ΔABC.

a) a point _____

b) a line segment _____

c) an angle _____

d) a vertex _____



2. Double the measure of each angle.

a) 14° _____ **c)** 73.3° _____

b) 121° _____ **d)** 164.8° _____

3. Determine the measure of each angle.

a) $\frac{1}{2}$ of 172° = _____ **b)** $\frac{1}{2}$ of 81° = _____

4. Calculate the measure of each angle.

a) 360° ÷ 2 = _____ **c)** 360° ÷ 6 = _____

b) 360° ÷ 60 = _____ **d)** 360° ÷ 12 = _____

5. Determine the measure of each unknown angle.

a) 37° + _____ = 90° **b)** 96° + _____ = 180°

6. Match each type of angle with its description.

right angle

straight angle

acute angle

obtuse angle

reflex angle

an angle less than 90°

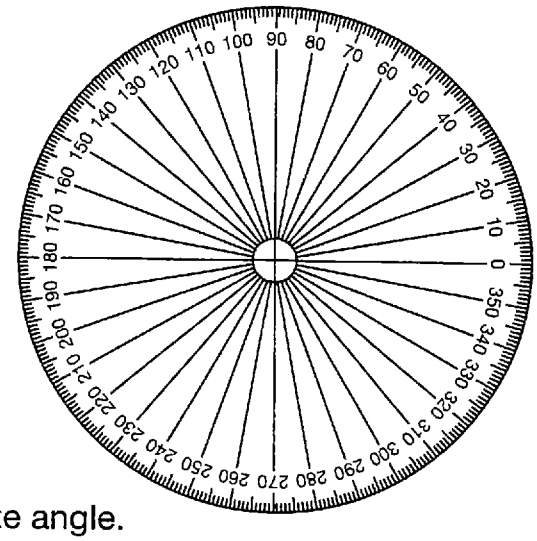
a 90° angle

an angle greater than 90° but less than 180°

a 180° angle

an angle greater than 180° but less than 360°

Use the circular protractor diagram to answer Questions 7 to 9.



7. Determine the number of degrees in each.

a) 1 whole circle _____

b) $\frac{1}{2}$ of a circle _____

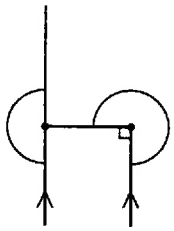
c) $\frac{1}{4}$ of a circle _____

8. Draw a 25° angle on the circular protractor. Mark the arc and label it 25°.

9. On the circular protractor, draw the hour and minute hands of a clock to show 10:00 p.m. Mark the arc and label the measure of the acute angle.

10. What three angle measures are shown in the chair diagram?

_____



11. Sketch each pair of line segments.

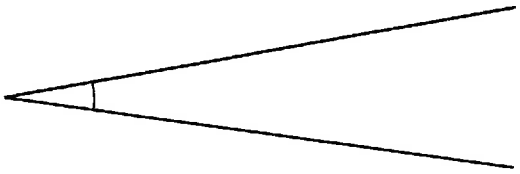
a) a pair of line segments that are **parallel** but are *not* vertical or horizontal

b) a pair of line segments that are **perpendicular** but are *not* vertical or horizontal

parallel
always the same distance apart

perpendicular
at right angles (90°)

12. a) Measure this acute angle using a protractor. _____



b) Determine the measure of the reflex angle. _____

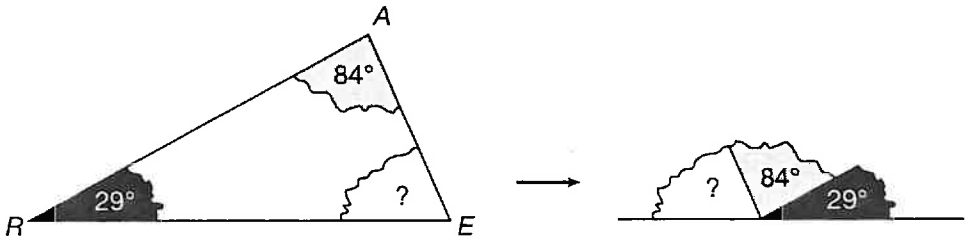
7.1 Estimating and Measuring Angles

You will need
- square paper
- a protractor

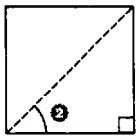
Try These

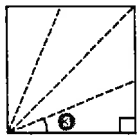
i) The 3 angles in ΔARE form a straight line, which measures _____.

ii) What is the measure of $\angle E$? 180° – _____ – _____ = _____



You can tell the measure of some angles without measuring.





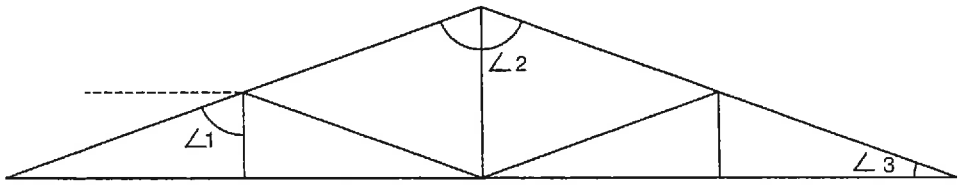
1. What is the angle measure of a square corner of paper? _____
2. What angle measure do you get when you fold a square along its diagonal? _____
3. What angle measure do you get when you fold the square so that the diagonal meets the base? _____

REFLECTING
What referents for angles do you use?

You can use **referents** like the angles above to estimate the measures of other angles.

Example 1

Estimate the measure of each angle marked on the sketch of the roof truss. (The first one is done for you.)



∠1: a bit less than 90° ∠2: _____ ∠3: _____

Estimate: about 70° Estimate: _____ Estimate: _____

Solution

A. Draw a dotted line to show how close the angle is to 90° or 180°.

B. Estimate the angle measure by comparing it with the referent angle you drew in Part A.

Example 2

What is the measure of the reflex angle at the peak of the roof truss?

Solution 1

A. Extend an arm to form a straight angle.

B. Measure the acute angle in the arc using a protractor and add it to 180°.

180° + _____ = _____

Solution 2

A. Measure the obtuse angle using a protractor. _____

B. Subtract that measure from the total number of degrees around a point.

360° – _____ = _____

REFLECTING

Do you prefer Solution 1 or Solution 2? Why?

Example 3

The elevation angle of a solar panel on a house should be between 25° and 70°. Albert, a building contractor, wants to install solar panels on a roof at an angle of 55°. Draw a 55° angle for the roof.

Solution

A. Draw one arm of the angle.

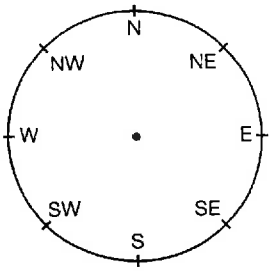
B. Use a protractor to locate a point on the other arm. Draw this arm.

C. Mark the arc and label the angle measure.

Hint

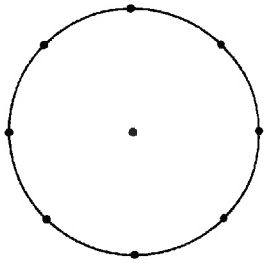
The centre mark of the protractor should be on the vertex. The baseline of the protractor should be on one arm.

Practice

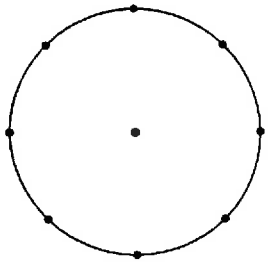


1. Sketch the smallest angle for each move of the needle on a compass. Label the angle measure.

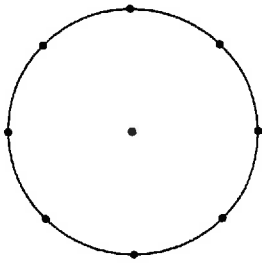
a) from N to W



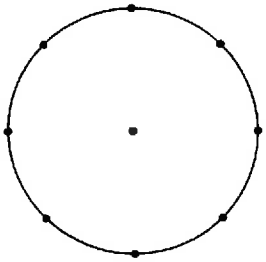
c) from NW to SE

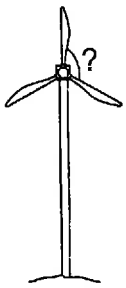


b) from E to SE



d) from S to NW



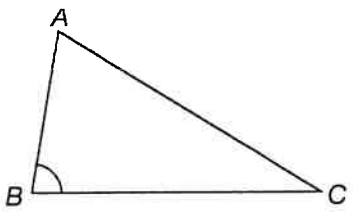


2. Calculate the measure of the equal angles between any two arms of a wind turbine. _____

3. Estimate the measure of $\angle ABC$ in each diagram. Draw dotted lines to show the referent angles you used.

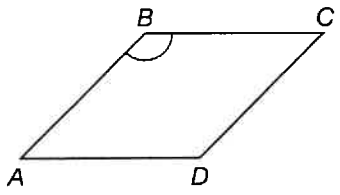
Hint
When three letters are used to name an angle, the middle letter identifies the vertex of the angle.

a)



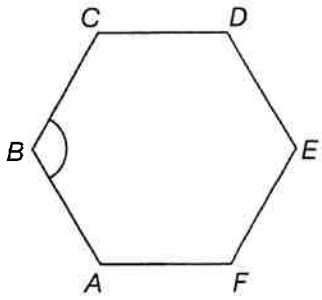
Estimate: _____

c)



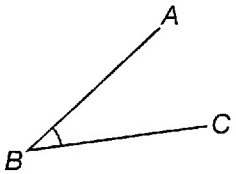
Estimate: _____

b)



Estimate: _____

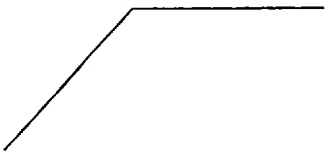
d)



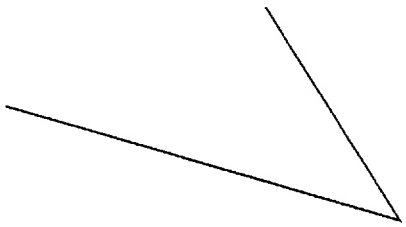
Estimate: _____

4. Measure and label the two angles at each vertex.

a)



b)



Hints

- What should the angle measures around each vertex add up to?
- Extend the arms so that the angle is large enough to measure.

5. Construct each angle.

a) 7°

b) 24°

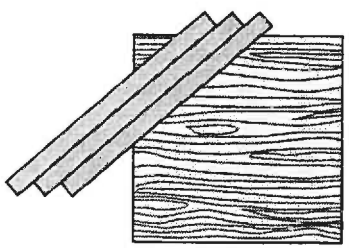
c) 51°

d) 133°

e) 272°

f) 315°

6. Sonya is building a square table. She wants to finish the top with wood veneer strips placed at a 45° angle to each edge. How can she do this without using a protractor?



7.2 Describing Angles

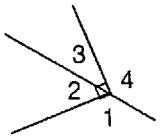
You will need
- a protractor

Try These

i) Which angles form a right angle? _____

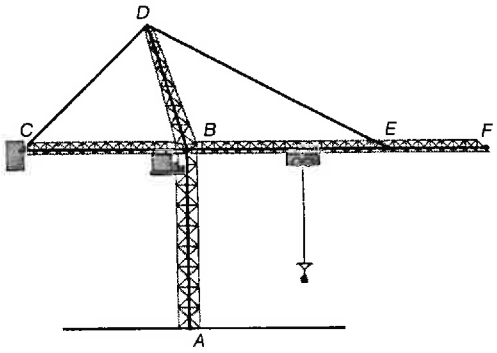
ii) Which angles form a straight angle? _____

iii) Which angles share a common side? _____



adjacent angles
angles that share a common vertex and a common arm

Angles are often described in pairs. In this sketch of a construction crane, $\angle ABC$ and $\angle CBD$ are **adjacent angles**.



1. Name another pair of adjacent angles.

_____

supplementary angles
two angles whose sum is 180°

2. Name two adjacent angles that are **supplementary**.

_____

3. Name two adjacent angles that are *not* supplementary angles.

_____

4. Connect C to A and E to A to form a right angle.

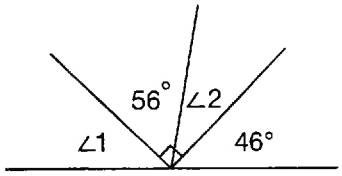
complementary angles
two angles whose sum is 90°

5. Name two adjacent angles that are **complementary**.

_____

Example

Calculate the measures of $\angle 1$ and $\angle 2$.



REFLECTING
How can you check your calculations using a protractor?

Solution

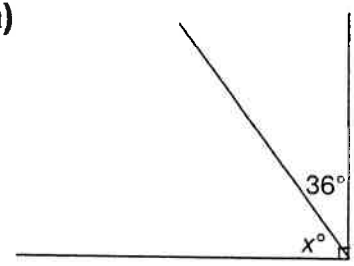
A. What is the measure of $\angle 2$? $90° - \_\_\_\_\_ = \_\_\_\_\_$

B. What is the measure of $\angle 1$? _____

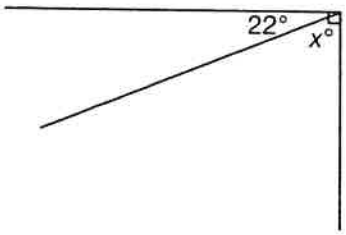
Practice

1. Label the missing angle measures in each diagram.

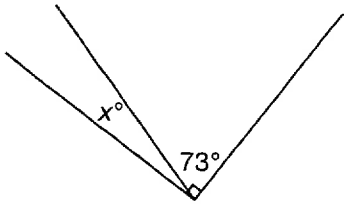
a)



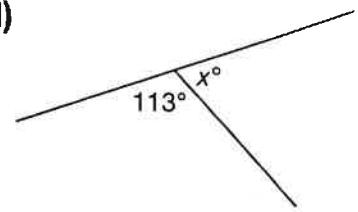
b)



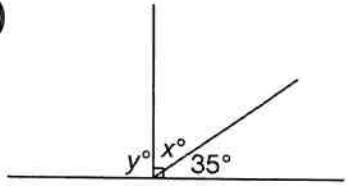
c)



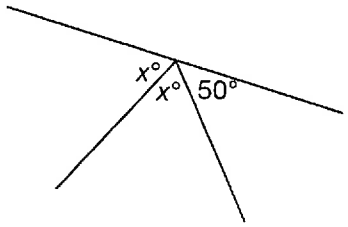
d)



e)



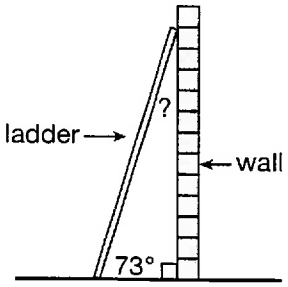
f)



2. On each diagram in Question 1, draw an adjacent angle that is *not* complementary or supplementary. Estimate the measure of each angle you drew.

3. Brad wants to attach trim at the top of a wall. The ladder, the ground, and the wall form a right triangle.

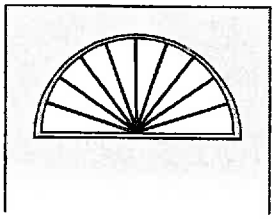
How can you calculate the measure of the angle formed between the top of the ladder and the wall?



4. Each angle around the centre of the window measures 18°.

a) What is the sum of all the 18° angles in the window?

b) How many of these angles would it take to make 90°?



Hint

An obtuse angle is greater than 90° but less than 180°.

7.3 Bisecting Angles

You will need
- tracing paper
- a protractor
- a compass

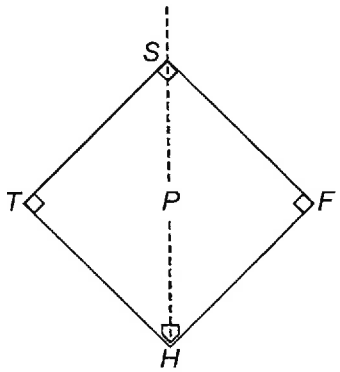
bisector
the line that divides an angle or line into two equal parts

Try These

i) half of 45° is _____

ii) double 51° is _____

In baseball, the pitcher's mound is located on the **bisector** of the angle at home plate, $\angle THF$. How can you bisect $\angle THS$ to locate the shortstop position?



1. using tracing paper:

_____

2. using a protractor: _____

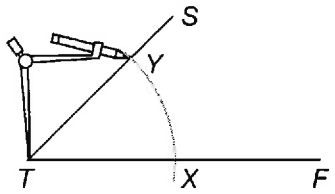
_____

Example

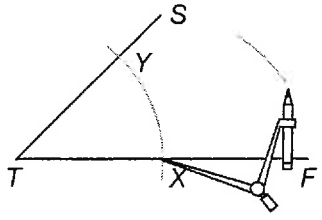
Use a compass and straightedge to bisect $\angle STF$.

Solution

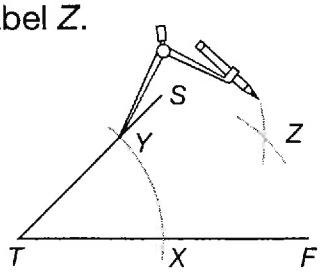
A. Draw an arc on $\angle STF$. Label X and Y.



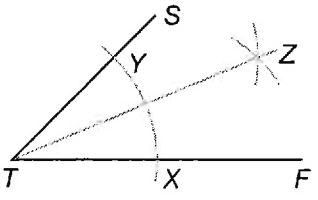
B. With centre X, draw an arc.



C. With centre Y and the same radius, draw another arc. Label Z.



D. Use a straightedge to join T to Z.

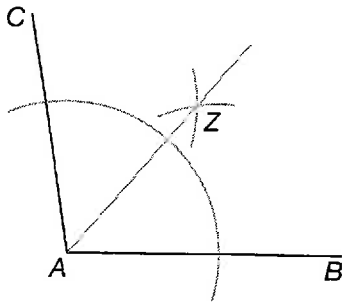


REFLECTING
Which method of bisecting an angle do you prefer? Why?

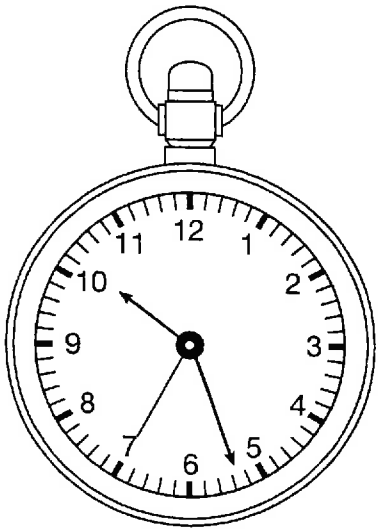
Practice

1. Pearl bisected obtuse $\angle CAB$.
What are the measures of the angles?

$\angle CAB$ _____ $\angle CAZ$ _____ $\angle ZAB$ _____



2. Shelly and Eric want to share a piece of pie. Draw an acute angle to represent the piece of pie. Then construct the bisector to create two equal pieces.



3. Matt looked at his watch and said, "The bisector of the reflex angle between the hour and minute hands would be located just before the 2."

   a) Do you agree with Matt? _____
   Check by bisecting the angle.

   b) Where is the bisector of the obtuse angle between the hour and second hands? Draw it.

4. Taylor says, "Bisecting an acute angle always results in two equal angles that are acute." Does bisecting an obtuse angle always result in two equal angles that are obtuse? Explain your thinking.

_____

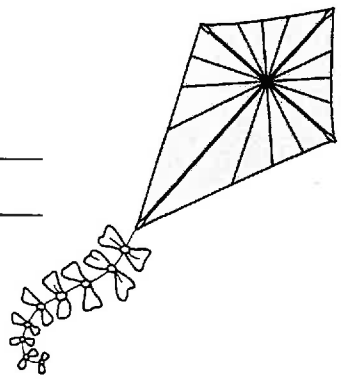
_____

5. The struts on this kite are **perpendicular bisectors.**
Where are bisected angles used in the kite?

_____

_____

_____



7.4 Replicating Angles

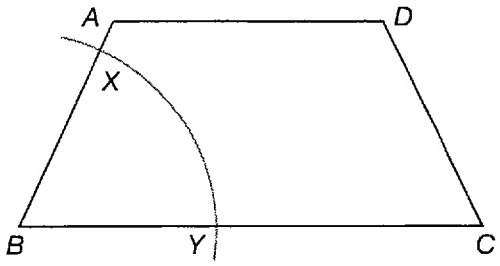
You will need
- a protractor
- a compass
- a straightedge

Try These

i) $\frac{180°}{360°} \times 100 =$ _____ % of circle

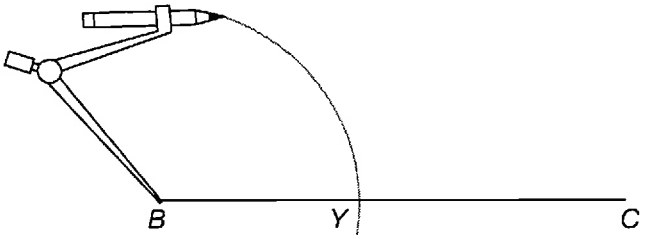
ii) $\frac{270°}{360°} \times 100 =$ _____ % of circle

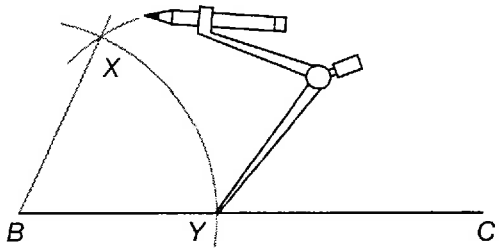
Ryan is making a cabinet with shelves of this shape. To copy acute $\angle B$, he uses the following method.



- Place a compass on vertex B and draw an arc on $\angle ABC$. Label the points of intersection X and Y.
- To start the copy, draw side BC. Then draw an arc on the copy with the same radius you used on the original. Label Y on the copy.
- Use the compass to measure XY on the original angle. Then draw an arc with that radius from point Y on the copy. Label X on the copy. Draw a line from B through X.

REFLECTING
Measure the original angle and the copied angle. How do they compare?





1. Use a compass and a straightedge to copy obtuse $\angle D$ in the trapezoid.

2. How can you copy the angle using only a protractor?

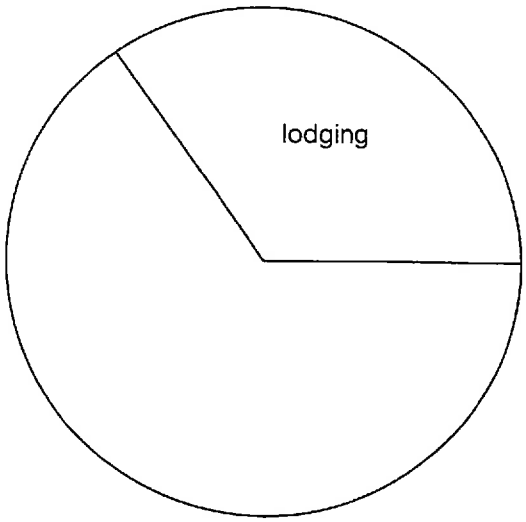
_____

_____

REFLECTING
Which method of copying an angle do you prefer? Why?

Example

Anna is using graphing software to make a pie chart of her budget. The chart represents the amount of money that goes for lodging, expressed as a percent of the total circle. Copy this angle and determine what percent of her budget goes for lodging.



Solution

A. Copy the angle for lodging, using a compass and a straightedge.

B. Is the measure of the copied angle equal to the measure of the original angle?

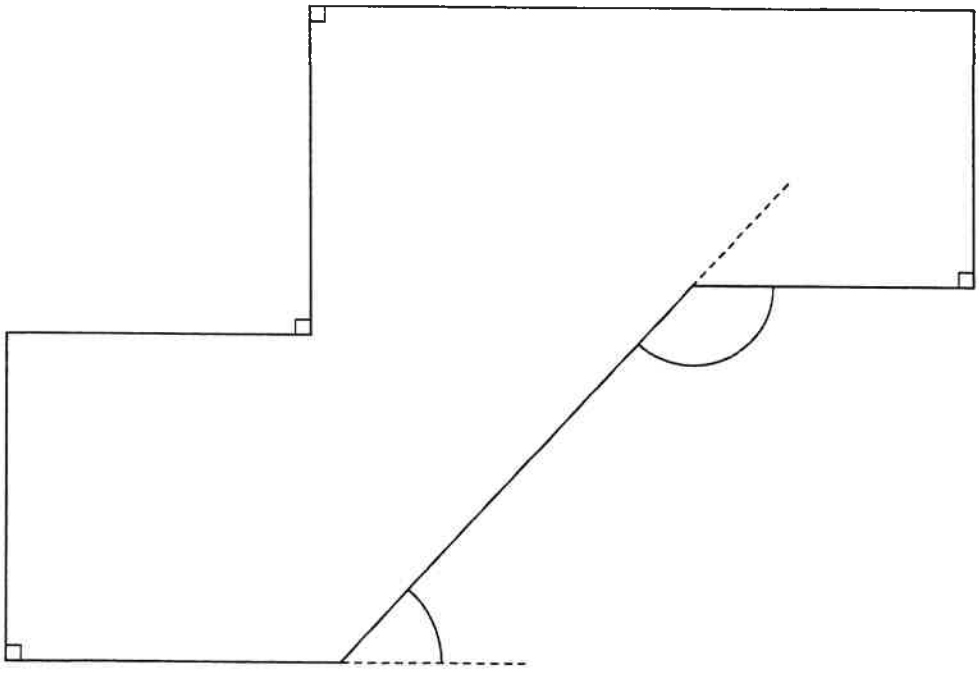
_____

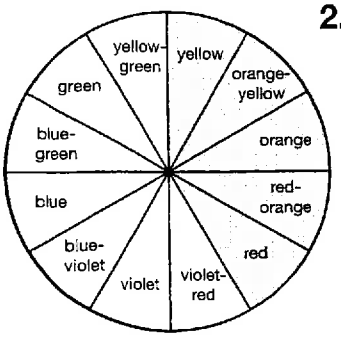
C. What percent of the circle is represented by lodging?

_____ × 100 ≐ _____%

Practice

1. Emily traced this plan for stairs from a book. Make a copy of the two marked angles under the stairs on a piece of paper.

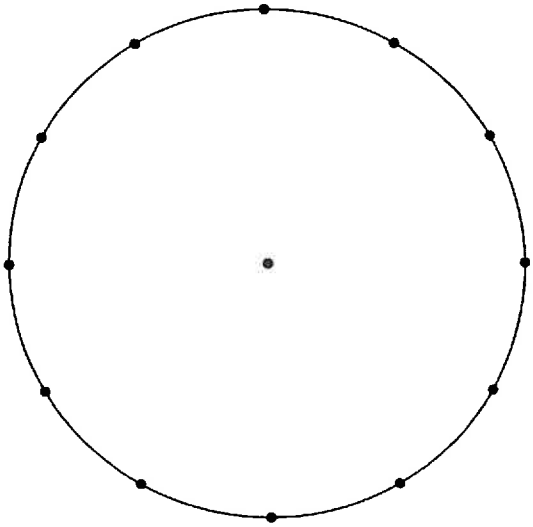




2. In art, the primary colours (red, yellow, and blue) are combined to make other colours. Stefan saw this colour wheel on the Internet; it shows 12 equal **sectors**.

a) Calculate the measure of each acute angle around the centre of the colour wheel.

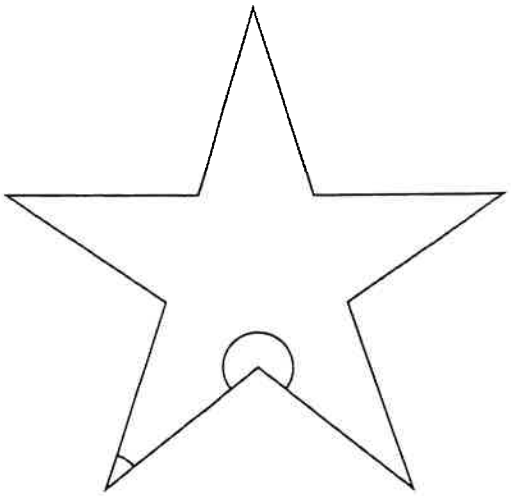
b) Use the circle diagram at the right to make a copy of the 12 equal sectors.



c) The "warm colours" extend from yellow to red. How many degrees of the circle is that?

d) What percent of the colour wheel is covered by the warm colours? (Round to one decimal place.)

3. Kathryn wants to construct a star logo for her sports team. Make a copy of each angle inside the star.



a) the acute angle

b) the reflex angle

Mid-Chapter Review

You will need
- a protractor
- a compass
- a straightedge

1. Name two of each, using letters.

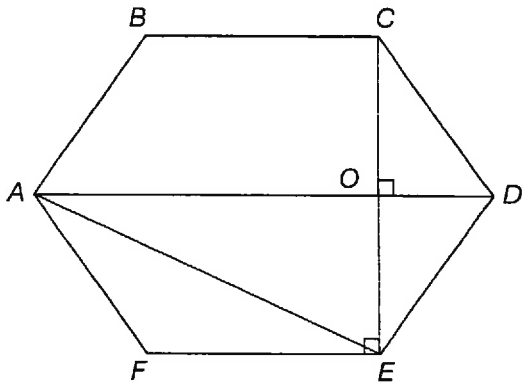
a) acute angles _____

b) obtuse angles _____

c) straight angles _____

d) complementary angles _____

e) supplementary angles _____



2. Use the diagram from Question 1.

a) Estimate the size of $\angle FAB$. _____

b) Mark an arc for each angle you named in Question 1 a), b), and d). Then measure and label each angle measure.

3. The grey areas represent the blind spots for a driver.

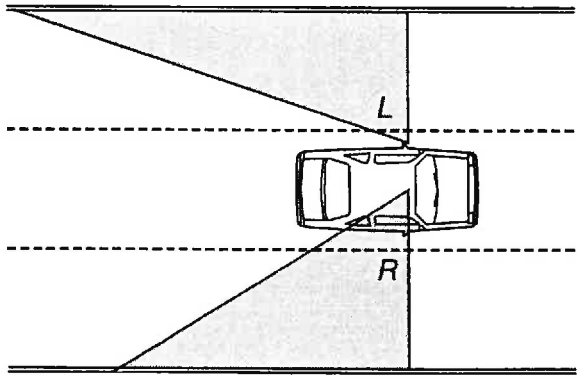
a) Estimate the size of $\angle L$ and $\angle R$.

$\angle L$ is about _____

$\angle R$ is about _____

b) On the diagram, bisect the larger blind-spot angle.

c) Make a copy of the smaller blind-spot angle, using a compass and a straightedge.



4. An equilateral triangle has sides of equal length and angles of equal measure. What is the measure of each angle? How do you know?

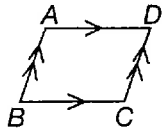
7.5 Classifying Lines and Angles

You will need
- a ruler
- a protractor
- a right triangle (optional)

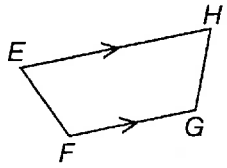
Try These

Name the parallel sides in each quadrilateral.

i)

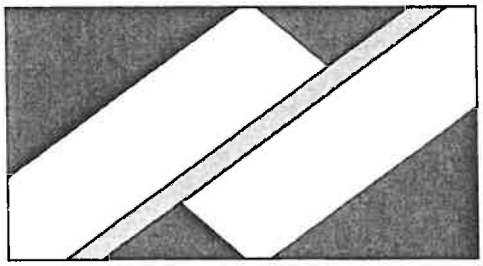


ii)



_____ _____

Many flags show parallel lines, which are always the same distance apart, and perpendicular lines, which are at right angles. This is the flag of the Franco-Yukonnais community—the French Canadian residents of Yukon. (The colours are blue, white, and yellow.)



1. Which lines inside the flag are parallel? Mark them using matching arrowheads.

Hints
- Use the symbol ∥ as a short way to write that a line is parallel to another line.
- Use the symbol ⊥ as a short way to write that a line is perpendicular to another line.

2. Label the corners of the rectangular flag *ABCD*. Then name two pairs of parallel sides. _____

3. Name two pairs of perpendicular sides.

_____

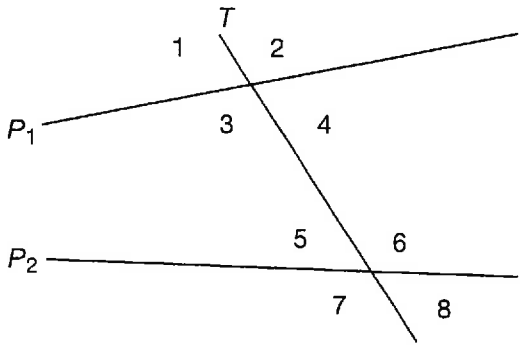
4. Complete the following statement: In a rectangle, the opposite sides are _____, and the adjacent sides are _____.

REFLECTING
Do perpendicular lines have to be horizontal and vertical? Use examples to explain.

5. Draw a flag that does *not* include parallel or perpendicular lines in its interior design.

Many angles are formed by two lines and a **transversal**. Below, lines P_1 and P_2 are intersected by T, a transversal.

transversal
a line that intersects two or more lines



6 Name all eight pairs of adjacent supplementary angles.

_____

_____

_____

corresponding angles
two angles formed by two lines and a transversal and located on the *same* side of the transversal

opposite angles
non-adjacent angles that are formed by two intersecting lines

alternate angles
two angles formed by two lines and a transversal and located on *opposite* sides of the transversal

Pairs of angles can be described in other ways. For example:

- **corresponding angles**: $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$ (These pairs are *above* or *below* lines P_1 and P_2.)
- **opposite angles**: $\angle 1$ and $\angle 4$, $\angle 2$ and $\angle 3$ (These are around an intersection point.)
- **alternate** interior **angles**: $\angle 3$ and $\angle 6$ (These pairs are *inside* lines P_1 and P_2.)
- **alternate** exterior **angles**: $\angle 1$ and $\angle 8$ (These pairs are *outside* lines P_1 and P_2.)

Example

What other pairs of corresponding angles, opposite angles, and alternate angles are in the diagram above?

Solution

A. corresponding angles: _____

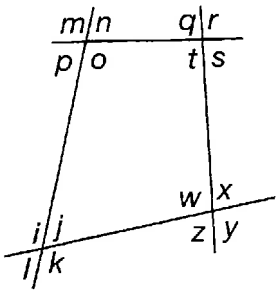
B. opposite angles: _____

C. alternate interior angles: _____

D. alternate exterior angles: _____

Hint
Look on the opposite sides of the transversal, *inside* lines P_1 and P_2.

Practice



1. Four lines intersect to form a quadrilateral.

a) Name a pair of opposite angles that are obtuse.

_____

b) Name two pairs of corresponding angles.

_____

c) Name two pairs of alternate interior angles.

_____

d) Name two pairs of alternate exterior angles.

_____

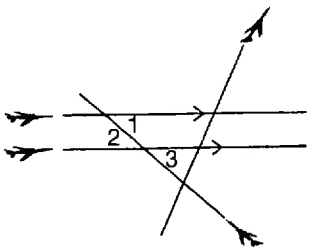
Hint
Use a ruler and a protractor to check.

2. In the diagram for Question 1, are any lines parallel or perpendicular? How do you know?

_____

_____

_____



3. Four stunt pilots passed in the air and the jet trails formed two parallel lines and two transversals.

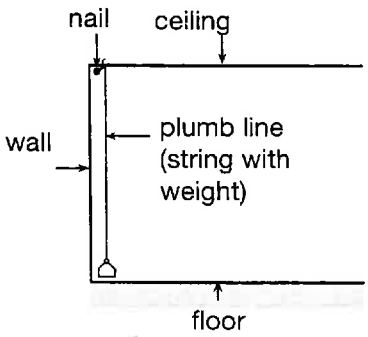
a) Describe the angles shown on the diagram.

$\angle 1$ and $\angle 2$: _____

$\angle 1$ and $\angle 3$: _____

b) How many pairs of opposite angles are there? _____

4. Christina is wallpapering a room. She uses a plumb line to mark the line where the first strip of wallpaper will be placed. This ensures that the wallpaper will be vertical even if the wall is crooked. Describe the parallel and perpendicular lines.



_____

_____

_____

_____

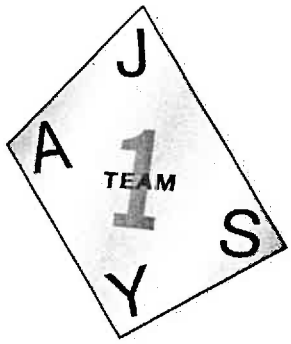
_____

5. Bruno is building a shed. How can he be sure that the ceiling and the floor are parallel? (Give at least two different ways.)

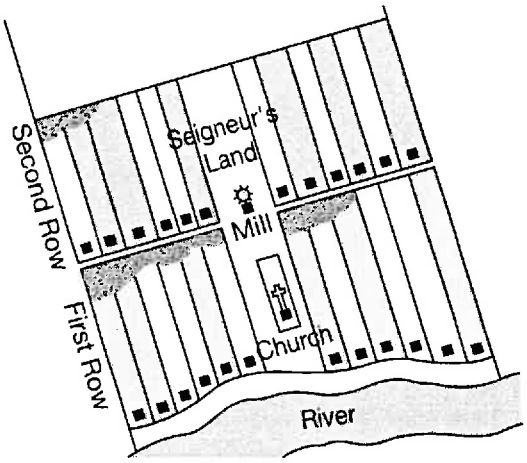
Hint

If two lines are perpendicular to a third line, then the two lines are parallel.

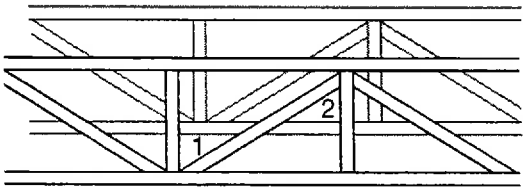
6. Name all the pairs of parallel lines and perpendicular lines in the fridge magnet shown at right.



7. The seigneurial system was a way of distributing plots of land in New France from 1627 to 1857. Land was surveyed close to a river because it was the main transportation route at that time. How does this system use parallel and perpendicular lines?



8. Floor joists like this are built in a new house to ensure that the floor surface is strong.



a) What type of angles are $\angle 1$ and $\angle 2$?

b) If the horizontal beams in the joists are parallel, what do you know about these angles?

7.6 Parallel Lines and Transversals

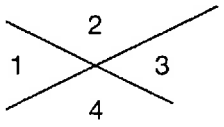
You will need
- a protractor
- a compass
- tracing paper

Try These

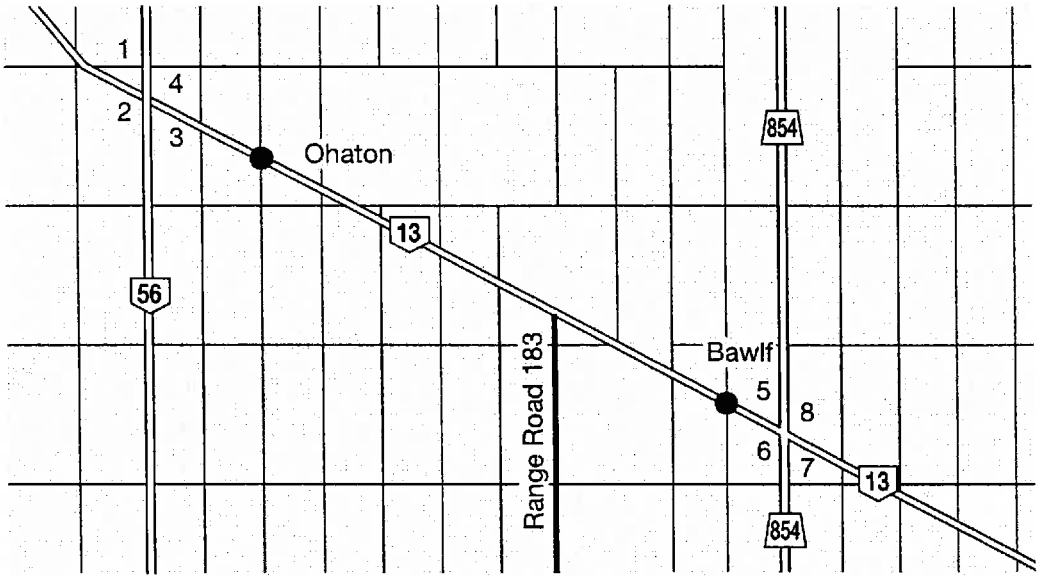
i) Name a pair of opposite angles. _____

ii) Name a pair of adjacent supplementary angles.

_____



This map of northern Alberta shows that Highway 56 and Highway 854 are parallel. Highway 13 intersects both of them.



Hint
Two corresponding angles form an F shape: F, ㄷ, ㅓ, or ㄱ.

1. Name pairs of corresponding angles labelled on the diagram.

_____

2. Compare the measures of the corresponding angles in each pair. What do you notice?

_____

Hint
Two alternate interior angles form a Z shape: Z, N, Σ, or И.

3. Name the pairs of alternate angles labelled on the diagram.

alternate interior angles: _____

alternate exterior angles: _____

4. Compare the measures of the alternate angles in each pair. What do you notice?

_____

5. Measure $\angle 4$ and $\angle 5$, the interior angles on the same side of the transversal. What do you notice?

_____

6. Measure the exterior angles $\angle 1$ and $\angle 8$ on the same side of the transversal. What do you notice?

_____

Hint
Two interior angles on the same side of a transversal form a C pattern: ⊏, ⊔, ⊐, or ⊓.

Example 1

How can you determine if Range Road 183 is parallel to Hwy 854?

Solution

A. Are the measures of the corresponding angles equal?

_____

B. The corresponding angles are _____ so the roads must be _____.

C. What is the relationship between corresponding angles and parallel lines? If a pair of corresponding angles are _____, then the lines are _____ OR If the lines are _____, then the corresponding angles are _____.

D. What is the relationship between alternate angles and parallel lines? If a pair of alternate angles are _____, then the lines are _____. OR If the lines are _____, then the alternate angles are _____.

E. What is the relationship between interior angles and parallel lines? If a pair of interior angles are _____, then the lines are _____. OR If the lines are _____, then the alternate angles are _____.

REFLECTING
What other pairs of angles could you measure with a protractor to determine if the roads are parallel?

Hint
Use the words equal, parallel, or supplementary to complete the sentences in Parts B to E.

Example 2

At the intersection of Hwy 854 and Hwy 13, there are two pairs of opposite angles. What can you say about the measures of opposite angles?

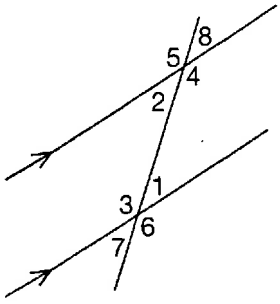
Solution

What are the measures of opposite angles?

_____

_____

Practice



1. a) State the alternate angles that are equal.

_____

_____

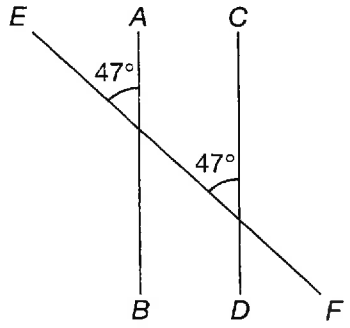
b) State the corresponding angles that are equal.

_____

_____

2. In each diagram, is *AB* parallel to *CD*? Explain how you know.

a)

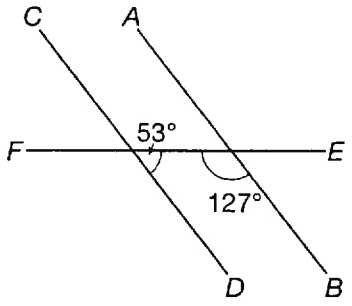


_____

_____

_____

c)

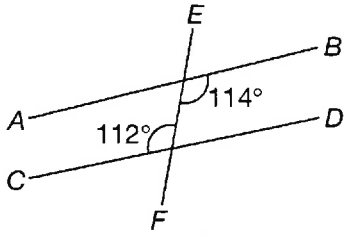


_____

_____

_____

b)

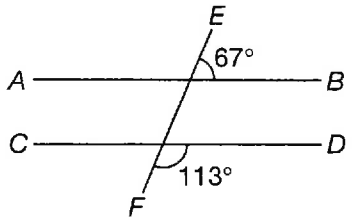


_____

_____

_____

d)

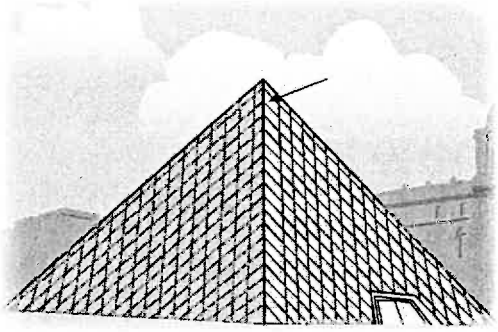


_____

_____

_____

3. The pyramid of the Louvre Museum (in Paris, France) was constructed using parallel lines. How can you determine the size of the marked angle in the top window by measuring another angle?



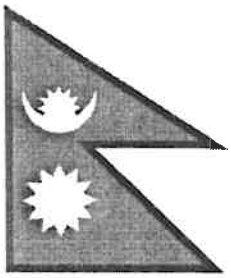
_____

_____

_____

_____

4. The flag of Nepal is unusual because it is not rectangular.



a) Trace and extend the two parallel lines and a transversal on the flag.

b) Mark a pair of alternate interior angles with dots. What is the relationship between these angles?

_____

c) Mark a pair of corresponding angles with arcs. What is the relationship between these angles?

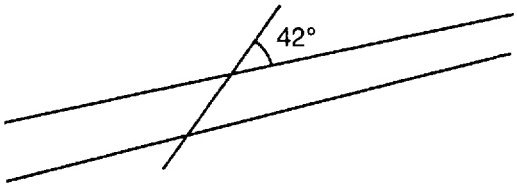
_____

d) Mark a pair of perpendicular lines with a little square. How do you know they are perpendicular?

_____

_____

5. This diagram shows a transversal crossing two lines.



a) What angle measures do you know without measuring?

_____

b) Can you conclude that the lines are parallel? Explain.

_____

_____

_____

7.7 Calculating Angles

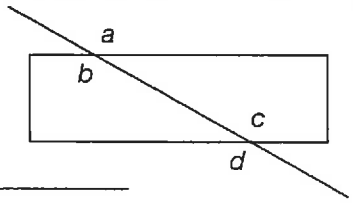
Try These

A transversal passes through a rectangle.

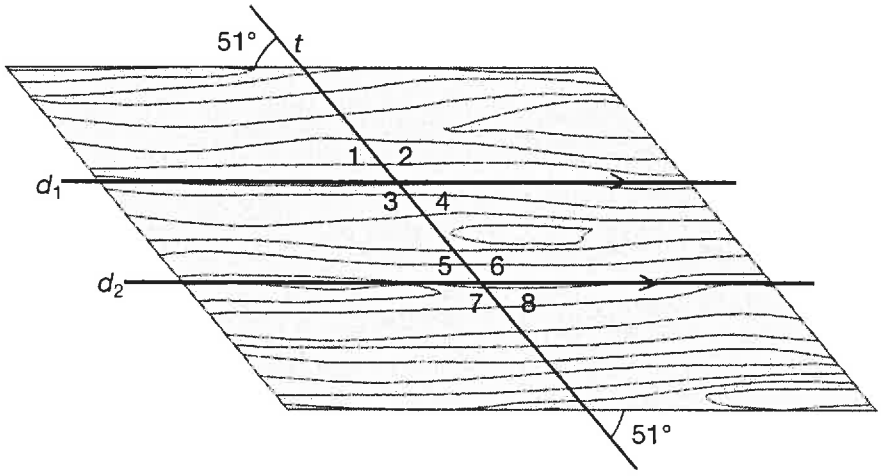
i) Name two corresponding angles.

_____

ii) Name two alternate exterior angles. _____



Edie cut some strips of flooring using two horizontal parallel cuts and one on a 51° angle.



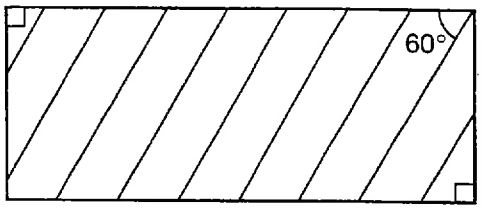
REFLECTING
What other ways can you see to reach the same answers?

1 What is the measure of each angle? Explain your thinking. The first one is done for you.

∠1 and ∠5 are 51°. These angles and the 51° angle at the top are corresponding angles formed by a transversal and parallel lines.

Example

This design in a rectangular floor shows many parallel and perpendicular lines. Determine the measures of the angles. Explain your thinking.



Solution

A. What is the measure of the eight acute angles across the *top* of the diagram? How do you know?

_____

_____

B. What is the measure of the eight acute angles across the *bottom*? How do you know?

_____

_____

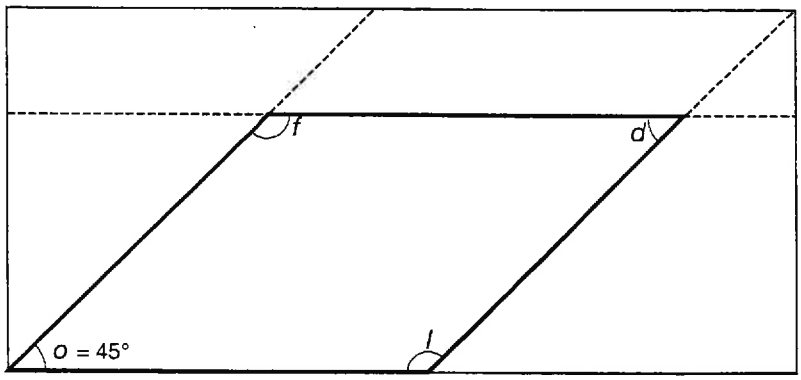
C. What is the measure of the seven obtuse angles across the bottom? How do you know?

_____

_____

Practice

1. Marty folded a rectangular piece of paper to form a parallelogram. He labelled the vertices *f, o, l, d* and measured one of the angles, $\angle o = 45°$.



a) Mark the parallel sides of the parallelogram.

b) Determine the measures of the other angles in the parallelogram. Explain your thinking.

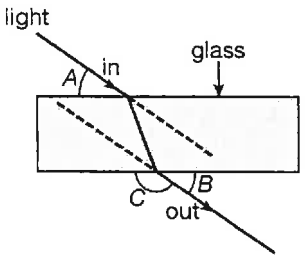
_____

_____

_____

c) On the diagram, label the measures of the angles outside the parallelogram.

2. Light refraction can be represented by two parallel lines and a transversal.



a) If $\angle A$ measures 32°, determine the measure of $\angle B$. Explain your thinking.

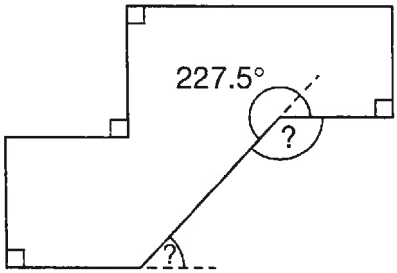
_____

_____

b) Determine the measure of $\angle C$. Explain your thinking.

_____

3. What is the measure of the two angles in the stair diagram? Explain your thinking.



_____

_____

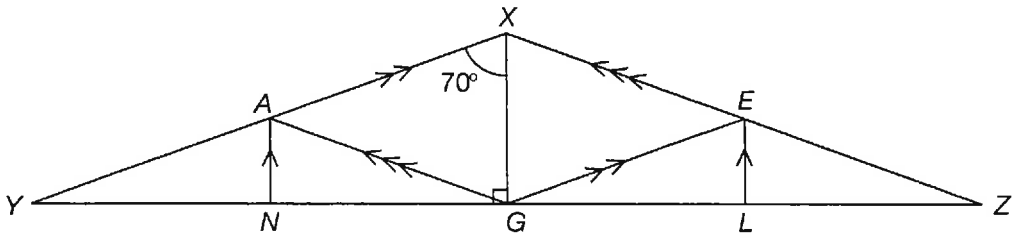
_____

_____

_____

_____

4. In this diagram of a roof truss, GX is the bisector of $\angle YXZ$. Calculate the measure of each angle below. Explain your thinking.



a) $\angle XGE$ _____

b) $\angle XYG$ _____

c) $\angle GXE$ _____

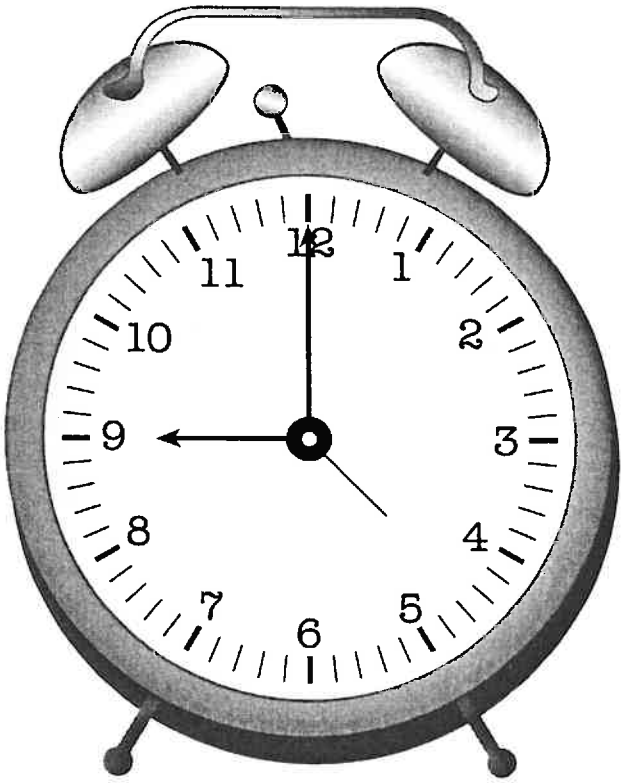
d) $\angle XGA$ _____

e) $\angle YGA$ _____

Solving a Clock Puzzle

7.8

Dan has an old wind-up alarm clock. At 9:00 p.m., he sets the alarm so that the alarm pointer bisects the reflex angle formed by the hour and minute hands. As he looks at the clock, he wonders how many times the hands form a 90° angle in a day.



A. What strategies can you use to figure out how many right angles are formed in 12 h?

B. Determine the number of times a right angle is formed in 24 h.

C. At what time did Dan's alarm clock go off the next morning? Explain how you know.

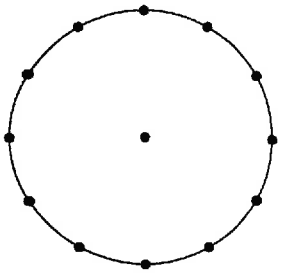
Chapter Review

You will need
- a compass
- a straightedge

1. Write three angle measures that are useful as referents when you estimate angles.

_____

Use the circle diagram to help you answer Questions 2 and 3.



2. Visualize a round pizza. What is the measure of each angle if the pizza is cut into each number of equal pieces?

a) 6 _____ **c)** 4 _____

b) 12 _____ **d)** 9 _____

3. Aidan cut a round pizza into eight equal pieces. Then he bisected one piece. What is the measure of each angle in the two smallest pieces?

4. Calculate the measure of each angle.

a) the complement of an angle whose measure is 23°

b) the supplement of an angle whose measure is 79°

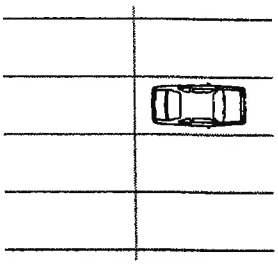
c) the third angle in a triangle whose other angles measure 35° and 66°

d) the reflex angle around a right angle

5. Sketch an angle for each type. Bisect the obtuse angle.

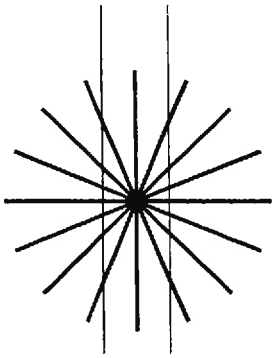
a) obtuse angle **b)** reflex angle

6. A parking lot shows five parallel lines with a transversal. If one angle measures 90°, what can you say about the measures of the other angles?

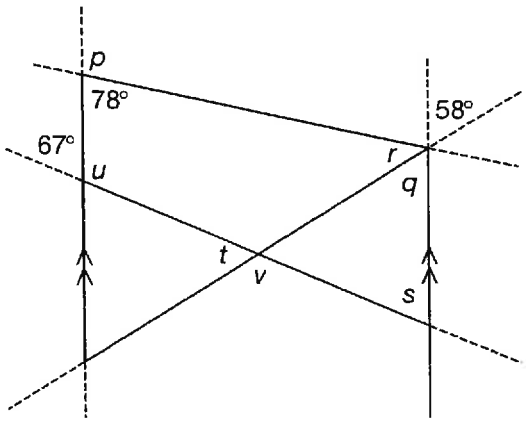


7. Karla is building a fence. She attached the top board to the first two posts. How can she be sure that the two posts are parallel to each other?

8. Brook says that the two lines in this optical illusion are not parallel. Do you agree or disagree? Explain your thinking.



9. Determine the measure of each angle. Explain your thinking.



a) $p =$

b) $q =$

c) $r =$

d) $s =$

e) $t =$

f) $u =$

g) $v =$

Chapter Test

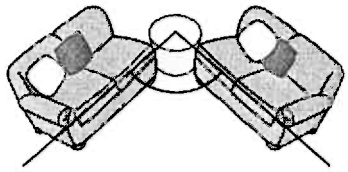
You will need
- a protractor
- a compass

1. Are the angles complementary, supplementary, or neither?

a) 34° and 56° _____ **c)** 99° and 81° _____

b) 23° and 163° _____ **d)** 74° and 16° _____

2. a) Estimate the measure of the obtuse angle shown.



_____

b) Bisect the obtuse angle.

c) Label the measure of each acute angle.

Hint
You can make a sketch for each part to help you answer Question 3.

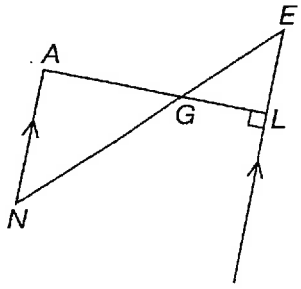
3. Visualize two lines that are intersected by a transversal. Tell if any of the lines are parallel, perpendicular, or neither for each condition given.

a) The corresponding angles are equal. _____

b) The interior angles on the same side of the transversal are 80°. _____

c) The alternate angles are right angles. _____

_____



4. a) Name a pair of parallel lines. _____

b) Name a pair of opposite angles. _____

c) Are $\angle ANG$ and $\angle LEG$ equal? Explain your thinking.

_____

d) What is the measure of $\angle NAG$? Explain your thinking.

_____

_____

5. Draw an angle of 20° using a protractor.

_____

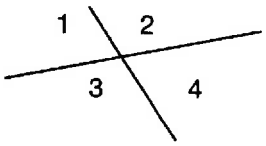
Glossary

A

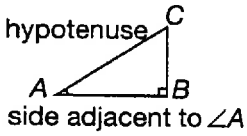
acres: a unit of measure for area in the imperial system
1 acre = 4840 sq yd
1 acre ≐ 0.405 ha

acute angle: an angle that measures more than 0° and less than 90°

adjacent angles: angles that share a common vertex and a common arm
For example, angles 1 and 2 are adjacent angles. Angles 3 and 4 are adjacent angles.

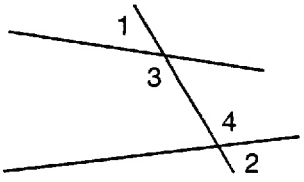


adjacent side: the side that is part of an **acute angle** in a **right triangle** but is not the **hypotenuse**
For example, AB is adjacent to $\angle A$.



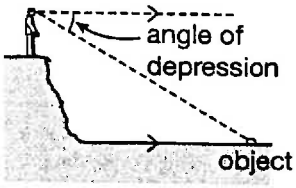
adjacent sides: two sides in a triangle or **polygon** that share a vertex

alternate angles: two angles formed by two lines and a **transversal** and located on opposite sides of the transversal
For example, angles 3 and 4 are alternate interior angles. Angles 1 and 2 are alternate exterior angles.

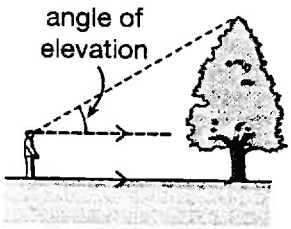


angle bisector: a line that cuts an angle in half to form two equal angles

angle of depression: the angle between the horizontal and the line of sight when looking down at an object



angle of elevation: the angle between the horizontal and the line of sight when looking up at an object



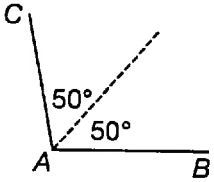
annual: for a year

B

base salary: payment for a given work period, such as an hour or a week, but not including additional pay

bisect: to divide into two equal parts

bisector: the line that divides an angle or line into two equal parts



bonus: an additional payment to a worker as a reward for meeting company goals

C

Canada Pension Plan (CPP): a government fund that provides a monthly pension to workers when they retire

capacity: the amount that a container can hold

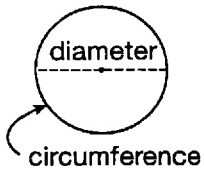
Celsius: a scale for temperature that includes the freezing point of water at 0° and the boiling point of water at 100°

centimetre (cm): a unit of measure for length in the metric system
1 cm = 10 mm
100 cm = 1 m

centre of rotation: a fixed point around which points in a shape are rotated. It can be inside or outside the shape.

charitable donations: an option for employees to make a regular donation to a charity

circumference: the perimeter of a circle
Circumference = $\pi \times d$, where d is the **diameter** (π is about 3.14)



commission: a payment based on a percentage of the worker's sales

company health plan: a plan for medical expenses not covered by other government health care plans

company pension plan: a fund that provides a company pension during retirement, in addition to **CPP**

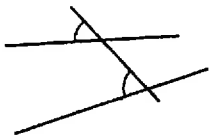
complementary angles: two angles whose sum is 90°

congruent: same size and shape

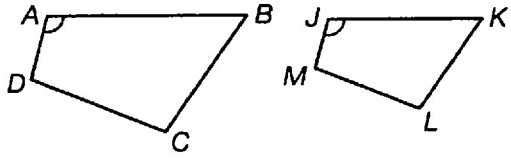
contract: a payment for a fixed period of time and/or a fixed amount of money

coordinates (x, y): a way to describe locations on a grid using a pair of numbers
For example, (−1, 3) lines up with −1 on the x-axis and 3 on the y-axis.

corresponding angles: **1.** two angles formed by two lines and a transversal and located on the same side of the transversal

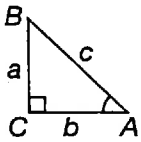


2. angles that match when two shapes are arranged to look the same



corresponding sides: sides that match when two shapes are arranged to look the same
For example, AB and JK are corresponding sides (above).

cosine: the ratio of the length of the **adjacent** leg to the length of the **hypotenuse** in a **right triangle**



$$\cos A = \frac{b}{c}$$

cup (c): a unit of measure for **capacity** in the imperial system
1 cup = 8 fluid ounces (US) or 10 fluid ounces (UK)
2 cups = 1 pint

D

decametre (dam): a unit of measure for length in the metric system
1 dam = 10 m
100 dam = 1 km

decimetre (dm): a unit of measure for length in the metric system
1 dm = 10 cm
10 dm = 1 m

diameter: a straight line through the centre of a circle that joins two points on the **circumference**
Diameter = **radius** × 2

dilation: the result of multiplying or dividing each length on a shape by the same number to create a similar shape

dilation centre: a fixed point from which a shape is enlarged or reduced

disability insurance: a plan that provides a source of income when an employee is injured and unable to work

double time: the hourly wage multiplied by 2

E

Employment Insurance (EI): a fund that provides income to people who lose their jobs (through no fault of their own) while they look for a new job

equilateral triangle: an equilateral triangle has equal sides and equal angles

F

face: a 2-D shape that forms a flat surface of a 3-D object

Fahrenheit: a scale for temperature that includes the freezing point of water at 32° and the boiling point of water at 212°

fluid ounce (fl oz): a unit of measure for **capacity** in the imperial system
1 fluid ounce = 2 tablespoons
8 fluid ounces = 1 cup (US) or
10 fluid ounces = 1 cup (UK)

foot (ft): an imperial unit of measurement for length
1 foot = 12 inches
3 feet = 1 yard

G

gallon (gal): a unit of measure for **capacity** in the imperial system
1 gallon = 4 quarts

gram (g): a unit of measure for **mass** in the metric system
1000 g = 1 kg

gross income: the total amount of money earned in a pay period before any deductions

H

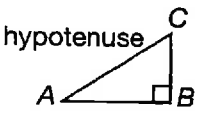
hectares (ha): a unit of measure for area in the metric system
1 ha is the same area as 1 square **hectometre**
1 ha = 1 hm^2

hectometre (hm): a unit of linear measure in the metric system
1 hm = 100 m
10 hm = 1 km

height: the perpendicular distance from the base of a **polygon** to an opposite vertex

hourly wage: a fixed payment for each hour of work

hypotenuse: the side of a **right triangle** that is opposite the 90° angle



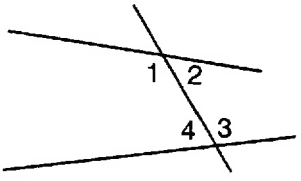
I

inch: an imperial unit of measurement for length
12 inches = 1 foot
36 inches = 1 yard

income: money received for work

income tax: a portion of a worker's earnings that federal and provincial governments use to provide services

interior angles: **1.** angles inside a polygon
2. angles between two lines
For example,



irregular polygon: a closed figure with straight sides with varying side lengths and angle measures

K

kilogram (kg): a metric unit of measure for **mass**
1 kg = 1000 g
1000 kg = 1 tonne (t)

kilolitre (kL): a unit of measure for **capacity** in the metric system
1 kL = 1000 L

kilometre (km): a unit of measure for length in the metric system
1 km = 1000 m

L

legs: the two sides that form the 90° angle in a **right triangle** (see **hypotenuse**)

life insurance: a plan that pays a sum of money to a family member or designated beneficiary in the case of an employee's death

line of reflection: the line across which a shape is flipped

litre (L): a metric unit of measure for **capacity**
1 L = 1000 mL
1000 L = 1 kL

M

mass: the amount of matter in an object. Common units of mass are **grams**, **kilograms**, and **tonnes** (metric) and **pounds** and **tons** (imperial).

metre (m): the base unit of measure for length in the metric system
1 m = 100 cm
1000 m = 1 km

midpoint: the point on a line segment that divides it into two equal parts

mile (mi): an imperial unit of measure for length
1760 yards = 1 mile
5280 feet = 1 mile

millilitre (mL): a metric unit of measure for **capacity**
1000 mL = 1 L

millimetre (mm): a unit of measure for length in the metric system
1000 mm = 1 m
10 mm = 1 cm

N

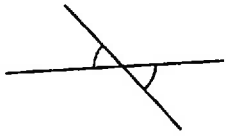
net: a composite 2-D shape that can be folded to create a 3-D object (such as a cube, cone, pyramid, cylinder)

net income: the money left after deductions are taken from gross income; also called take-home pay

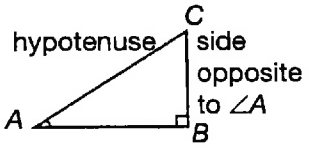
O

obtuse angle: an obtuse angle is greater than 90° but less than 180°

opposite angles: non-adjacent angles that are formed by two intersecting lines



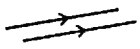
opposite side: the side that is directly across from a specific **acute angle** in a **right triangle**
For example, *BC* is opposite ∠*A*.



ounce (oz): a unit of measure for **mass** in the imperial system
16 ounces = 1 pound

P

parallel: two or more lines that are always the same distance apart

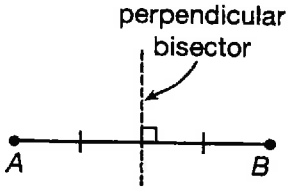


payroll savings: an option for employees to make a regular contribution to a savings plan, such as Canada Savings Bonds

perimeter: the distance around an object

perpendicular: two lines that form a right angle (90°)

perpendicular bisector: a line that bisects a line segment and is perpendicular to the line segment



pi (π): the ratio of the **circumference** of a circle to its **diameter**. Its value is about 3.14.

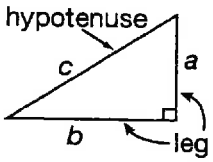
piecework: a payment based on the number of items created or completed

pint (pt): a unit of measure for **capacity** in the imperial system
1 pint = 2 cups
2 pints = 1 quart

polygon: a closed figure with straight sides

pound (lb): a unit of measure for **mass** in the imperial system
1 pound = 16 **ounces**
2000 pounds = 1 **ton**

Pythagorean theorem: a statement of a relationship in which the sum of the squares of the lengths of the **legs** of a **right triangle** is equal to the square of the length of the **hypotenuse**
$a^2 + b^2 = c^2$

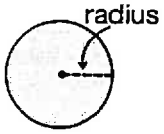


Q

quart (qt): a unit of measure for **capacity** in the imperial system
1 quart = 2 pints
4 quarts = 1 gallon

R

radius: a straight line from the centre of a circle to any point on the **circumference**



rate of exchange: the amount that money is worth from one currency to another. This varies daily.

ratio: a comparison of quantities with the same units

reciprocal: the multiplier of a number that gives 1 as a result
For example, the reciprocal of $\frac{1}{2}$ is $\frac{2}{1}$ or 2.
$\frac{1}{2} \times \frac{2}{1} = 1$ and
$1 \div \frac{1}{2} = \frac{2}{1}$

referent: a known measure used for comparing and estimating

reflection: the result of flipping a 2-D shape across a line

reflex angle: an angle that measures between 180° and 360°

regular polygon: a closed figure with all sides equal and all angles equal

right angle: an angle that measures 90°

right triangle: a triangle that contains a right angle

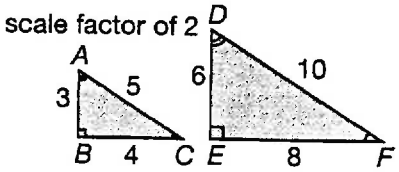
rotation: the result of turning a 2-D shape around a point. Rotations can go clockwise (cw) or counterclockwise (ccw).

royalty: a payment for a piece of work that is marketed and sold. The amount is based on a percentage of sales.

S

salary: a regular fixed payment for work, usually expressed as an amount per year but paid regularly (e.g., every two weeks or monthly)

scale factor: the number that the dimensions of a **polygon** are multiplied by to calculate the corresponding dimensions of a **similar polygon**

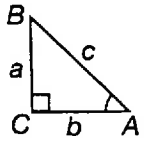


sectors: sections of a circle

shift premium: an additional amount of money for working outside of regular workday hours or on weekends

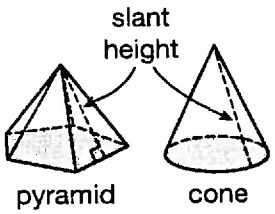
similar polygons: **polygons** that are congruent or are enlargements or reductions of each other. The ratios of corresponding lengths are the same, and corresponding angles are equal.

sine: the ratio of the length of the **opposite** leg to the length of the **hypotenuse** in a **right triangle**



$\text{Sin } A = \frac{a}{c}$

slant height: the distance from the top to the base, at a right angle, along a slanted side of a pyramid or cone. It is measured to the **midpoint** of the base side for a pyramid.



square number: the result when a whole number is multiplied by itself

straight commission: payment based only on sales made

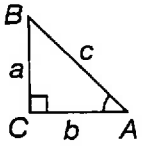
supplementary angles: two angles whose sum is 180°

surface area: the sum of all the areas of the faces of a 3-D object

symmetrical: a way of describing a shape that can be folded along at least one line so one half fits exactly over the other

T

tangent: the ratio of the length of the **opposite** leg to the length of the **adjacent** leg



$\text{Tan } A = \frac{a}{b}$

time and a half: the hourly wage multiplied by a factor of 1.5

ton (T): a unit of measure for **mass** in the imperial system
1 ton = 2000 pounds

tonne (t): a metric unit of measure for **mass**
1 t = 1000 kg

transformation: the result of moving or changing a shape according to a rule. The new shape is called the image.

translation: the result of sliding a 2-D shape along a straight line. On a grid, you can translate a shape right, left, up, or down.

translation rule: a way of describing a translation with numbers and directions
For example, "8 units right and 4 units up" or (R8, U4)

transversal: a line that intersects two or more lines

trigonometry: the study of relationships among the sides and angles in **right triangles**

U

union dues: a deduction made when an employee belongs to a union. Unions negotiate wages, benefits, and working conditions with employers.

unit price: the amount of money charged for a unit of an item

V

vertex: the point where two or more lines meet

volume: the amount of space occupied by a 3-D object

W

wage and tips: an hourly wage plus varying amounts in tips for services provided

Y

yard: an imperial unit of measure for length
1 yard = 3 feet
1 yard = 36 inches

Charts and Formulas

Metric Units

Length	Area	Volume	Capacity	Mass
kilometre (km) 1 km = 1000 m	square kilometre (km^2) 1 km^2 = 1 000 000 m^2 1 km^2 = 100 ha	cubic kilometre (km^3) 1 km^3 = 1 000 000 000 m^3	kilolitre (kL) 1 kL = 1000 L	kilogram (kg) 1 kg = 1000 g 1000 kg = 1 t
hectometre (hm) 1 hm = 100 m	square hectometre (hm^2) 1 hectare (ha) = 1 hm^2 1 ha = 10 000 m^2	cubic hectometre (hm^3) 1 hm^3 = 1 000 000 m^3	hectolitre (hL) 1 hL = 100 L	hectogram (hg) 1 hg = 100 g
decametre (dam) 1 dam = 10 m	square decametre (dam^2) 1 dam^2 = 100 m^2	cubic decametre (dam^3) 1 dam^3 = 1000 m^3	decalitre (daL) 1 daL = 10 L	decagram (dag) 1 dag = 10 g
metre (m) 1 m = 100 cm	square metre (m^2) 1 m^2 = 10 000 cm^2	cubic metre (m^3) 1 m^3 = 1 000 000 cm^3	litre (L) 1 L = 1000 mL	gram (g) 1 g = 1000 mg
decimetre (dm) 1 dm = 0.1 m	square decimetre (dm^2) 1 dm^2 = 0.01 m^2	cubic decimetre (dm^3) 1 dm^3 = 0.001 m^3	decilitre (dL) 1 dL = 0.1 L	decigram (dg) 1 dg = 0.1 g
centimetre (cm) 1 cm = 0.01 m 1 cm = 10 mm	square centimetre (cm^2) 1 cm^2 = 0.0001 m^2	cubic centimetre (cm^3) 1 cm^3 = 0.000 001 m^3 Note: 1 cm^3 holds 1 mL	centilitre (cL) 1 cL = 0.01 L	centigram (cg) 1 cg = 0.01 g
millimetre (mm) 1 mm = 0.001 m	square millimetre (mm^2) 1 mm^2 = 0.000 001 m^2	cubic millimetre (mm^3) 1 mm^3 = 0.000 000 001 m^3	millilitre (mL) 1 mL = 0.001 L	milligram (mg) 1 mg = 0.001 g

Imperial Units

Length	Area	Volume	Capacity	Mass
inch (in. or ")	square inches (sq in.)	cubic inches (cu in.)	tablespoon (T)	ounces (oz)
foot (ft or ') 1 foot = 12 inches	square feet (sq ft) 1 sq ft = 144 sq in.	cubic feet (cu ft) 1 cu ft = 1728 cu in.	fluid ounce (fl oz) 1 fl oz = 2 T	pound (lb) 1 lb = 16 oz
yard (yd) 1 yard = 3 feet	square yard (sq yd) 1 sq yd = 9 sq ft	cubic yard (cu yd) 1 cu yd = 27 cu ft	cup (c) 1 c = 8 fl oz (US) 1 c = 10 fl oz (UK)	ton (T) 1 T = 2000 lb (US) 1 T = 2240 lb (UK)
mile (mi) 1 mile = 1760 yd	square mile (sq mi) 1 sq mi = 3 097 600 sq yd	cubic mile (cu mi)	pint (pt) 1 pt = 2 c	
	1 acre = 4840 sq yd		quart (qt) 1 qt = 2 pt	
			gallon (gal) 1 gal = 4 qt	

Converting Common Imperial Units to Metric (SI)

Linear	Area	Volume	Capacity	Mass
1 in. $\doteq$ 2.54 cm	1 sq in $\doteq$ 6.4516 cm^2	1 cu in. $\doteq$ 16.39 cm^3	1 fl oz $\doteq$ 29.57 mL	1 oz $\doteq$ 28.35 g
1 ft $\doteq$ 0.31 m	1 sq ft $\doteq$ 0.0929 m^2	1 cu ft $\doteq$ 28.32 dm^3	1 pt $\doteq$ 0.47 L, or 470 mL	1 lb $\doteq$ 0.45 kg
1 yd $\doteq$ 0.91 m	1 sq yd $\doteq$ 0.8361 m^2	1 cu yd $\doteq$ 0.76 m^3	1 qt $\doteq$ 0.95 L, or 950 mL	1 T $\doteq$ 0.91 t
1 mi $\doteq$ 1.61 km	1 sq mi $\doteq$ 2.5900 km^2	1 cu mi $\doteq$ 4.17 km^3	1 gal $\doteq$ 3.79 L, or 3790 mL	
	1 acre $\doteq$ 0.4047 ha			

Converting Common Metric (SI) Units to Imperial

Linear	Area	Volume	Capacity	Mass
1 mm $\doteq$ 0.039 in.			1 mL $\doteq$ 0.03 fl oz	
1 cm $\doteq$ 0.39 in.	1 cm^2 $\doteq$ 0.1550 sq in.	1 cm^3 $\doteq$ 0.06 cu in.		
1 m $\doteq$ 1.09 yd	1 m^2 $\doteq$ 10.7639 sq ft	1 m^3 $\doteq$ 1.31 cu yd	1 L $\doteq$ 2.11 pt	1 g $\doteq$ 0.04 oz
1 m $\doteq$ 3.27 ft			1 L $\doteq$ 1.06 qt	1 kg $\doteq$ 2.21 lb
1 km $\doteq$ 0.62 mi	1 km^2 $\doteq$ 0.3861 sq mi	1 km^3 $\doteq$ 0.24 cu mi	1 L $\doteq$ 0.26 gal	1 t $\doteq$ 1.10 T

Temperature

$$F = \frac{9}{5}C + 32$$

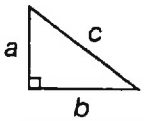
$$C = \frac{5}{9}(F - 32)$$

Circle Formulas

Diameter = radius × 2
Circumference
$= \pi \times$ diameter
Circumference
$= \pi \times$ radius × 2
Area: $\pi \times r^2$

Pythagorean theorem

$a^2 + b^2 = c^2$, where a and b are sides adjacent to the right angle in a right triangle and c is the hypotenuse



Primary Trigonometric Relationships

$$\sin A^\circ = \frac{\text{opposite side of } A^\circ}{\text{hypotenuse}}$$

$$\cos A^\circ = \frac{\text{adjacent side of } A^\circ}{\text{hypotenuse}}$$

$$\tan A^\circ = \frac{\text{opposite side of } A^\circ}{\text{adjacent side of } A^\circ}$$